

Accelerated math. Homework 5.



Problems marked with * are more difficult.

1. Compute:

$$\begin{array}{ll} 1) \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}; & 4) 1\frac{1}{2} \cdot 1\frac{1}{3} \cdot 1\frac{1}{4} \cdot 1\frac{1}{5}; \\ 2) \frac{6}{7} \cdot \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} \cdot \frac{10}{11}; & 5) \left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right) \cdot \left(1 + \frac{1}{7}\right) \cdot \left(1 + \frac{1}{8}\right); \\ 3) \frac{1}{2} \cdot \frac{2}{3} \cdot \dots \cdot \frac{23}{24} \cdot \frac{24}{25}; & 6) \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right) \cdot \dots \cdot \left(1 - \frac{1}{99}\right) \cdot \left(1 - \frac{1}{100}\right). \end{array}$$

$$\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot 5} = \frac{1}{5}$$

$$\frac{6}{7} \cdot \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} \cdot \frac{10}{11} = \frac{6 \cdot \cancel{7} \cdot \cancel{8} \cdot \cancel{9} \cdot \cancel{10}}{\cancel{7} \cdot \cancel{8} \cdot \cancel{9} \cdot \cancel{10} \cdot 11} = \frac{6}{11}$$

$$\frac{1}{2} \cdot \frac{2}{3} \cdot \dots \cdot \frac{23}{24} \cdot \frac{24}{25} = \frac{1 \cdot \cancel{2} \cdot \dots \cdot \cancel{23} \cdot \cancel{24}}{\cancel{2} \cdot \cancel{3} \cdot \dots \cdot \cancel{24} \cdot 25} = \frac{1}{25}$$

$$1\frac{1}{2} \cdot 1\frac{1}{3} \cdot 1\frac{1}{4} \cdot 1\frac{1}{5} = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \frac{6}{5} = \frac{3 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 4 \cdot 5} = \frac{6}{2} = 3$$

$$\left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right) \cdot \left(1 + \frac{1}{7}\right) \cdot \left(1 + \frac{1}{8}\right) = \frac{5}{4} \cdot \frac{6}{5} \cdot \frac{7}{6} \cdot \frac{8}{7} \cdot \frac{9}{8} = \frac{9}{4}$$

$$\left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right) \cdot \dots \cdot \left(1 - \frac{1}{99}\right) \cdot \left(1 - \frac{1}{100}\right) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \dots \cdot \frac{98}{99} \cdot \frac{99}{100} = \frac{1}{100}$$

2. Fill in the missing number to have the right equalities.

$$\frac{2}{5} \cdot \underline{\hspace{1cm}} = 1$$

$$2\frac{1}{2} \cdot \underline{\hspace{1cm}} = 1$$

$$1 : \frac{2}{5} = \underline{\hspace{1cm}}$$

$$1 : \underline{\hspace{1cm}} = \frac{7}{4}$$

$$\underline{\hspace{1cm}} \cdot \frac{12}{11} = 1$$

$$\underline{\hspace{1cm}} \cdot 1\frac{1}{3} = 1$$

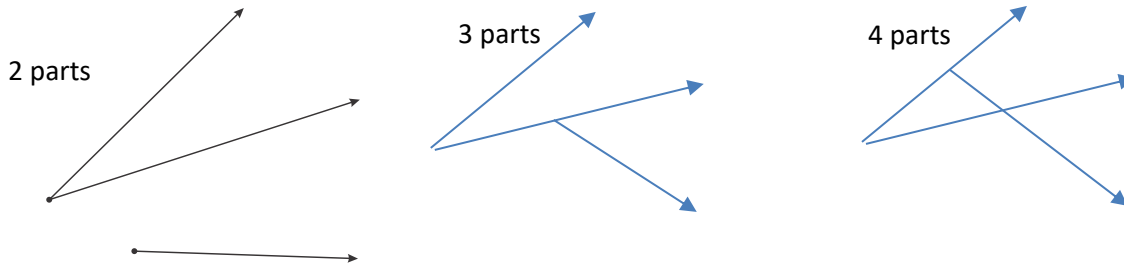
$$1 : \frac{12}{11} = \underline{\hspace{1cm}}$$

$$1 : \underline{\hspace{1cm}} = \frac{3}{10}$$

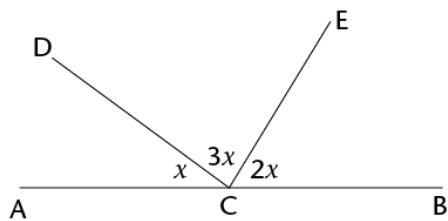
$$\frac{2}{5} \cdot \frac{5}{2} = 1, \quad 2\frac{1}{2} \cdot x = \frac{5}{2} \cdot \frac{2}{5} = 1, \quad 1 \div \frac{2}{5} = \frac{5}{2}, \quad 1 \div \frac{4}{7} = \frac{7}{4}$$

$$\frac{11}{12} \cdot \frac{12}{11} = 1, \quad \frac{3}{4} \cdot \frac{4}{3} = 1, \quad 1 \div \frac{12}{11} = \frac{11}{12}, \quad 1 \div \frac{10}{3} = \frac{3}{10}$$

3. Into how many parts do 3 rays on the picture below divide a plane? Draw 3 rays in a way that they divide the plane into 3 parts, 4 parts, do not divide a plane into parts. (Any 2 points in the same part can be connected without crossing the edge, not necessarily by a straight line)



4. A (natural) number which is less than 30 upon division by 2, 3, and 4 gives the remainder 1. What is this number? (Find all possible solutions).
If we subtract 1 from this number, we will get a number which is divisible by 2, 3, and 4. We have only 2 such numbers which are less than 30. They are 12 and 24, so our numbers are 13 and 25.
5. Calculate the measure of angle x from the picture below (points A, C and B lie on the same line)



$$x + 3x + 2x = 180$$

$$6x = 180$$

$$x = 180 \div 6 = 30$$

Answer: 30°

6. Fill up the table:

a	5	2	-8	-8	$-(-189)$	43
$-a$	-5	-2	8	8	-189	-43

7. Pencils are packed into big and small boxes. In 4 big and 3 small boxes there are 132 pencils, in 2 big and 3 small boxes there are 84 pencils. How many pencils are there in one small box?

Given:	solution
In 1 big box $\rightarrow x$ pencils In 1 small box $\rightarrow y$ pencils	Difference between 132 and 84 pencils is coming from 2 big boxes, so

$4x + 3y = 132$ $2x + 3y = 84$	$2x = 132 - 84 = 48$ $x = 24$ $2 \cdot 24 + 3x = 84$ $48 + 3y = 84$ $3y = 36$ $y = 36 \div 3 = 12$
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8. 4 little ducklings and 5 little geese weight 4 kg and 100 g. 5 little ducklings and 4 little geese weight 4 kg. How much does one little goose weight?

given	solution
1 duckling $\rightarrow x$ g 1 goose $\rightarrow y$ g $4x + 5y = 4100$ g $5x + 4y = 4000$ g	4 little ducklings, 5 little geese weight, 5 little ducklings, and 4 little geese weight 8100 g (8 kg 100 g) altogether, so we can right the following equation: $9x + 9y = 8100$ $9(x + y) = 8100$ $x + y = 900$ g $4x + 5y = 4x + 4y + y = 4(x + y) + y = 4100$ $4 \cdot 900 + y = 4100$ $y = 4100 - 4 \cdot 900 = 500$ g, $x = 900 - 500 = 400$ g Answer: One little goose weight 500 g.

9. Solve the following equations:

$x + \frac{4}{5} = \frac{9}{10}$ $x + \frac{4}{5} - \frac{4}{5} = \frac{9}{10} - \frac{4}{5}$ $x = \frac{9}{10} - \frac{4}{5} = \frac{9}{10} - \frac{8}{10}$ $x = \frac{1}{10}, \quad \frac{1}{10} + \frac{4}{5} = \frac{9}{10}$	$y - \frac{4}{9} = \frac{5}{6}$ $y = \frac{5}{6} + \frac{4}{9}$ $y = \frac{15}{18} + \frac{8}{18} = \frac{23}{18}$ $\frac{23}{18} - \frac{4}{9} = \frac{23}{18} - \frac{8}{18} = \frac{15}{18} = \frac{5}{6}$	$\frac{1}{2}z + \frac{3}{4} = \frac{3}{2}z - \frac{1}{4}$ $\frac{3}{4} + \frac{1}{4} = \frac{3}{2}z - \frac{1}{2}z$ $z = 1$ $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = \frac{3}{2} - \frac{1}{4}$
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10. Simplify the following expressions:

a. $2 + 3a + xy + 4 - a + xy - 6 = 2a + 2xy$

b. $d - 4 + t + t + 32 + 3d = 4d + 2t + 28$

c. $x + 5s - 3s + 2x = 3x + 2s$

11. On the first shelf there are 5 more books than on the second shelf and 5 less than on the third shelf. There are 105 books altogether. How many books are there on each shelf? (Write an equation to solve the problem.)

given	solution
First shelf $\rightarrow x$ books	$x + (x - 5) + (x + 5) = 105$
Second shelf $\rightarrow x - 5$	$x + x + x + 5 - 5 = 3x = 105$
books	$x = 105 \div 3 = 35$
Third shelf $\rightarrow x + 5$	Answer: 1 st shelf – 35 books, 2 nd shelf – 30 books, 3 rd shelf 40
Altogether $\rightarrow 105$ books	books.

12.

$$2(4 + 9w) = 8 + 18w$$

$$(2 - 5m) \cdot (-5) = -10 + 25m$$

$$-8(6x + 3) = -48x - 24$$

$$4(-6z + 4) = -24z + 16$$

$$-4(-4d - 5) = 16d + 20$$

$$-9(n - 4) = -9n + 36$$

$$-6(8p + 3) = -48p - 18$$

$$(-5d + 1)(-2) = 10d - 2$$

$$2(3v - 8) = 6v - 16$$

$$-4(9k + 9) = -36k - 36$$