## Algebra.

## 1. Positive and negative numbers. Absolute value of a number.

Coordinates are a set of values that show an exact position of an object. How many variables do we need to find our place in a theater? In a plane? What we can use as values?


How many different variables do we need to find our position on the surface of the Earth?

How many values do we need to show the exact position of the point on a number line?

Find the coordinates of points A, B, C, D, E, F, G, and H on the number line below:


Mark the points $\mathrm{A}(0), \mathrm{B}(1), \mathrm{C}\left(-1 \frac{1}{2}\right), \mathrm{D}(5), \mathrm{E}(-5), \mathrm{F}(-3), \mathrm{G}(3)$


Is there anything in common between points F and G , points D and E ?
On a straight line we can't say anything about the position of each point, we need to have a refence, point of origin, and we also need to know the length of a unit segment. After we choose the position of 0 and the length of the unit segment we can find a very important information about each point, the distance between the point and the origin, 0 . By choosing the direction of
increase of numbers, we define the sign (positive or negative) of all numbers. For each point on a number line we can assign a number, negative or positive, and each of this number can be described by the distance from 0 and the sign. This distance is called an absolute value of a number. And, of cause, it is always positive.

$$
\left\{\begin{array}{lr}
|a|=a, & \text { if } a \geq 0 \\
|a|=-a, & \text { if } a<0
\end{array}\right.
$$

$$
|5|=\quad|-5|=\quad|10|=\quad|-10|=
$$

Does a fraction have an absolute value?

$$
\left|\frac{1}{2}\right|=\quad\left|-\frac{1}{2}\right|=
$$

Can we solve the following equation? How many solutions does it have.

$$
|x|=5
$$

To solve an equation means to find all possible values which will give us a true statement when put into the equation instead of a variable.

$$
|x|=3 \quad|y|=10 \quad|z|=-2
$$

1. Write the coordinates and absolute values of the points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}$ marked on the number lines below:

2. A swimming pool can be filed by one pipe in 10 hours or by another pipe in 15 hours. How long it will take to fill up the pool with both pipes opened?
3. A swimming pool can be filed with one pipe in 10 hours. Full pool can be drain out with another pipe in 20 hours. How long it will take to fill up the pool with opened drain pipe?
4. Compare:

| $\|7+3\| \quad\|7\|+\|3\|$ | $\|7-3\| \quad\|7\|-\|3\|$ |
| :---: | :---: |
| $\|7-3\| \quad\|3-7\|$ | $\|3-7\| \quad\|3\|-\|7\|$ |
| $\|a-b\| \quad\|b-a\|$ | $\|7-3\| \quad\|7\|+\|3\|$ |
| $\|3 a\| 3 *\|a\|$ | $\|a+b\| \quad\|a\|+\|b\|$ |
| $\|b * a\| \quad b *\|a\|$ |  |

Now let's try to add 2 numbers, one positive and one negative. (Subtraction is always can be seen as an addition of a negative number; subtraction of a negative number, as we all know, is an addition of a positive number.) As an example we will add 115 and ( -75 ) and 75 and $(-115)$. In both cases absolute values of summands are the same, but in the first case the absolute value of the negative number is smaller than that of the positive summand and in the second case absolute value of the negative summand is greater. Look at the picture:

a) represents absolute values of $|115|=|-115|,|70|=|-70|$ and their difference $|115|-|70|$. Another visual representation of the same absolut valus is shown on the picture below.

$$
\begin{aligned}
& |115|=|-115| \bullet \\
& |70|=|-70| \bullet
\end{aligned}
$$

b) represents subtraction of 70 ( or addition of -70 ) from (to) 115 . If we subtract (or add negative) number which absolute value is smaller from the one which absolute value is grater we will do the usual operation of subtraction as we did before when we only operated with natural numbers.
c) represents the operation of subtraction (or addition of a negative number) of a number which absolute value is grater then the one of the number from which we are subtracting. The absolute value of the result will be exactly the same, as in previous example, but the result itself will be the negative number, opposite to the result in the previous example.
5. Compute:
a. $465-283=$
b. $253-465=$
c. $89-121=$
d. $121-89=$

## Geometry.

$5^{\text {th }}$ postulate:


2 lines are crossed by transversal. If 2 consecutive interior angles formed by this transversal are supplementary (add up to a straight angle), then these 2 lines are parallel. (if they add up to an angle less than straight angle, these 2 lines will intersect on this side). Also, if two lines are
parallel, 2 consecutive interior angles are supplementary. Base on this postulate, a few theorems can be proved:

1. If 2 parallel lines crossed by transversal, consecutive angles are equal.
$\angle 1=\angle 6, \angle 7=\angle 3$, and so on.
Proof:
$\angle 2+\angle 1=180$ (straight angle) by $5^{\text {th }}$ postulate, $\angle 2+\angle 6=180$ (straight angle) $\Rightarrow \angle 6=\angle 1$
(Converse theorem are also can be proved:
If consecutive angles formed by transversal are equal, two lines are parallel)
2. If 2 parallel lines crossed by transversal, alternate angles are equal.
$\angle 2=\angle 3, \angle 1=\angle 8$.
Proof:
$\angle 2+\angle 1=180$ (straight angle) by $5^{\text {th }}$
postulate, $\angle 1+\angle 3=180$ (straight
angle) $\Rightarrow \angle 2=\angle 3$

Also, the theorem of the angles of a triangle:

Three angles of any triangle sum to a straight angle.
Line $l$ is parallel to line AC. Angles (3) are equal as vertical angles, angles (2) are equal and angles (1) are equal because line $l$ is parallel to line AC.


Proof:
Let's draw a line 1, parallel to the one side of the triangle. Based on the previous theorems, proved above, we can see that three angles ad up to a straight angle.


