

Algebra.

1.

$(2x - 4) - (13x + 3) = 15$	$-7y - 28 = 5y + 32$
$\frac{1}{3}x + 4 = 20 - \frac{1}{6}x$	$12 - 7x = 16x + 3$

2.

a. $m - (n + m) =$

b. $-(n - x) - x =$

c. $p + (-m + k - p) =$

$$d. -a - (m - a + p) =$$

$$e. -(m - a) - (k + a) =$$

$$f. m + (k - a - m) =$$

$$g. m - (a + m) - (-a - m) =$$

3.

$$|x - 5| = 4$$

$$\left| \frac{2}{5}x + \frac{1}{5} \right| = 10$$

$$||2x - 1| - 4| = 6$$

4. Peter got a new book. On the first day he read $\frac{1}{3}$ of the whole book and on the second day he read $\frac{1}{3}$ of the rest of the book. On the third day, Peter once again read $\frac{1}{3}$ of the rest of the book and now he needs to read 80 more pages to finish the book. How many pages are there in the book?
5. Jane and Mary are doing fall clean up in a backyard. Mary can do the job in 6 hours; together they can do it in 4 hours. How many hours does Jane need to clean up the backyard?

6. 5 hamsters will eat 5 bags of hamster food in 5 days. How many days 10 hamsters need to eat 10 bags of food?
7. A farmer has a cow, a goat and a goose. The cow and the goat will eat all the grass on his meadow in 45 days, the cow and the goose will eat all the grass on the same meadow in 60 days, and the goat and the goose will eat all the grass on the meadow in 90 days. How many days will it take them altogether to eat all the grass on the meadow? (we assume that the new grass is not growing.)

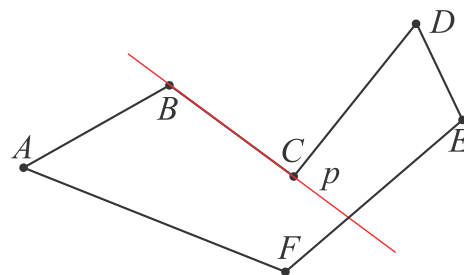
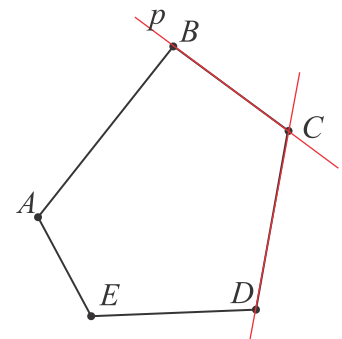
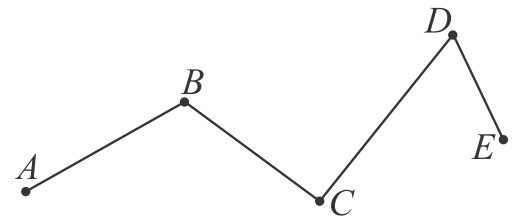
Geometry.

By combining several segments of a line together, so that the end of one segment is the beginning of the next, and any two adjacent segments do not lie on one line, we will create a figure that can be called a "broken" line. If the end of the last segment is the beginning of the first one, then the figure is a closed figure (area inside of the figure is a closed area), a polygon. It has vertices and sides. Polygons can be convex or concave. If the all vertices of the polygon are on the one side of any straight line containing any of the side or on that line, the polygon is called a convex polygon.

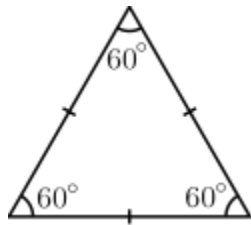
Otherwise the polygon is called concave.

Any two points inside of convex polygon can be connected by a segment without crossing the side of the polygon, for the concave polygon this is not the case.

Minimal number of vertices for polygon is 3, such polygon is called a triangle.



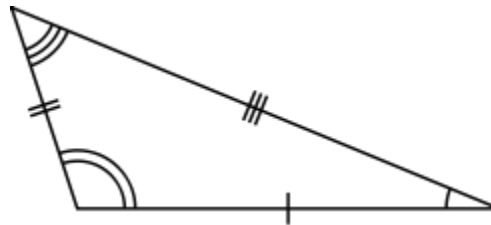
Classification of the triangles by their sides:



Equilateral

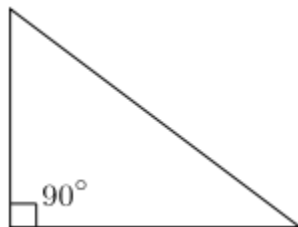


Isosceles

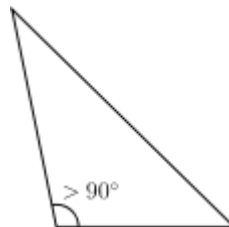


Scalene

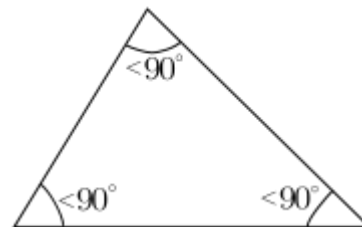
or by their angles:



Right



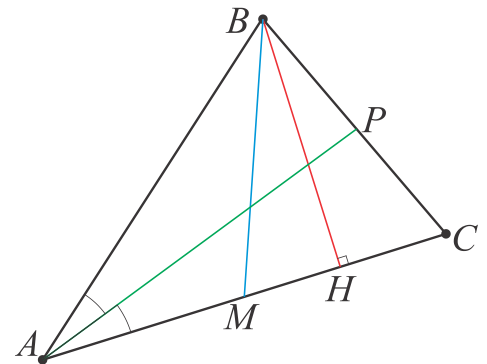
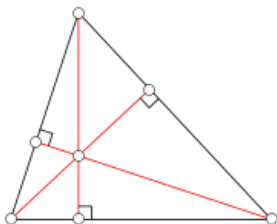
Obtuse



Acute

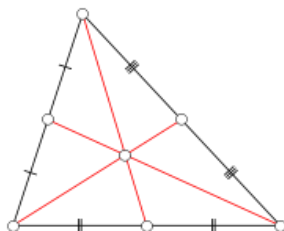
Special segments of a triangle.

From each vertex of a triangle to the opposite side 3 special segments can be constructed.

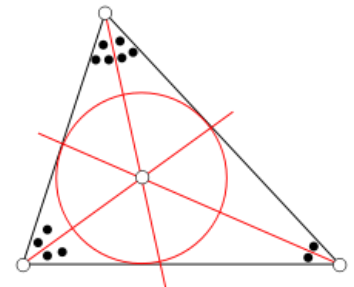


An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.

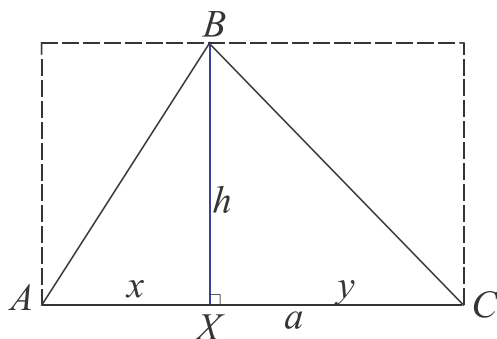
An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.



A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.



Area of a triangle.



$$S_{\Delta} = \frac{1}{2} h \times a$$

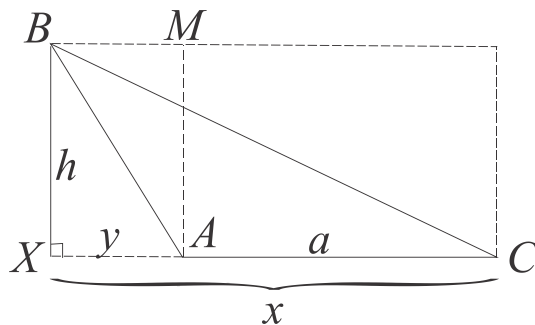
The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

$$S_{\square} = h \times a = x \times h + y \times h$$

$$S_{\Delta ABX} = \frac{1}{2} h \times x, \quad S_{\Delta XBC} = \frac{1}{2} h \times y, \quad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

$$S_{\Delta ABC} = \frac{1}{2} h \times x + \frac{1}{2} h \times y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$



For an obtuse triangle, for one out of the three heights, it is not so obvious.

$$S_{\Delta XBC} = \frac{1}{2} h \times x, \quad S_{\Delta XBA} = \frac{1}{2} h \times y$$

$$\begin{aligned} S_{\Delta ABC} &= S_{\Delta XBC} - S_{\Delta XBA} = \frac{1}{2} h \times x - \frac{1}{2} h \times y \\ &= \frac{1}{2} h \times (x - y) = \frac{1}{2} h \times a \end{aligned}$$