

Algebra.

1.

Simplify the following fractions:

a. $\frac{am+5a}{m+5},$

c. $\frac{49+84}{77},$

b. $\frac{5kx-5xa}{k-a},$

d. $\frac{35b}{7ba+7bx},$

2. A farmer has a cow, a goat and a goose. The cow and the goat will eat all the grass on his meadow in 45 days, the cow and the goose will eat all the grass on the same meadow in 60 days, and the goat and the goose will eat all the grass on the meadow in 90 days. How many days will it take them altogether to eat all the grass on the meadow? (we assume that the new grass is not growing.)

Exponent.

Exponentiation is a mathematical operation, written as b^n , involving two numbers, the **base** b and the **exponent** n . When n is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, b^n is the product of multiplying n bases:

$$b^n = \underbrace{b \times \cdots \times b}_n$$

In that case, b^n is called the n -th power of b , or b raised to the power n .

Properties of exponent:

If the same base raised to the different power and then multiplied:

$$b^3 \times b^4 = (b \times b \times b) \times (b \times b \times b \times b) = b \times b \times b \times b \times b \times b \times b = b^{3+4} = b^7$$

Or in a more general way:

$$b^n \times b^m = b^{n+m}$$

If the base raised to the power of n then raised again to the power of m :

$$(b^2)^3 = (b \cdot b)^3 = (b \cdot b) \cdot (b \cdot b) \cdot (b \cdot b) = b^{2 \cdot 3} = b^6$$

$$(b^n)^m = b^{n \cdot m}$$

$$b^1 = b;$$

$$b^0 = 1, \text{ for any } b \text{ except } 0.$$

If two different bases raised to the same power, then:

$$(a \cdot b)^3 = (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) = a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 b^3$$

$$(a \cdot b)^n = a^n b^n$$

The exponent indicates how many copies of the base are multiplied together. For example,

$3 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

$$2^3 \cdot 2^2, \quad 5^2 \cdot 5,$$

Rewrite the following expression without parenthesis:

$$\left(\frac{1}{2} + a\right)(2 + a) =$$

$$(n - a)(n + a) =$$

$$(a + b)(a + b) = (a + b)^2 =$$

$$(2a + 2b)(b - c) =$$

How to multiply one expression by another expression?

$$(a + b) \cdot (c + d) = ?$$

We know how to multiply an expression by a number using the distributive property: $a \cdot (b + c) = ab + ac$. What should we do to multiply one expression by another? To simplify the problem let's do the substitution, $a + b = u$ and use the distributive property:

$$(a + b) \cdot (c + d) = u(c + d) = uc + ud$$

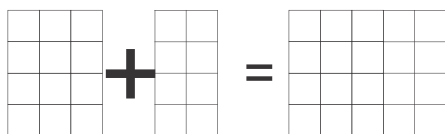
This new expression is not exactly the result what we are looking for; so, we need to put back $(a + b)$ instead of u :

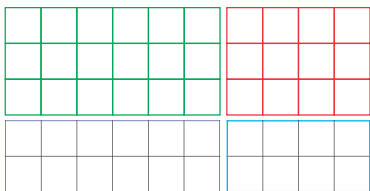
$$uc + ud = (a + b)c + (a + b)d$$

To get the final result let's use the distributive property again:

$$(a + b)c + (a + b)d = ac + bc + ad + bd$$

Here are the illustrations of the distributive properties:





$$4 \times 3 + 4 \times 2 = 4 \times (3 + 2)$$

$$(6 + 4)(2 + 3) = 6 \times 2 + 6 \times 3 + 4 \times 2 + 4 \times 3$$

Can we multiply expressions with more terms, like

$$(a + b + c)(d + m + n),$$

$$(2a + 2b)(b - c)$$

Factorize (represent as a product of two or more factors):

$$\frac{3}{4}x + \frac{3}{4}y =$$

$$5a - a^2 =$$

$$-3x - 3m =$$

Simplify the following expression:

$$\frac{2}{3} + 2x \left(\frac{1}{2} - \frac{1}{3}y \right) - x - \frac{1}{3}(2 - 2xy) =$$

Geometry.

Problems about triangles:

Perimeter.



Perimeter is the distance around a two dimensional shape, a measurement of the distance around something; the length of the boundary.

1. Find the perimeter of a triangle with sides
 - a. 4 cm, 5 cm, and 6 cm.
 - b. 2 cm, 3 cm, and 6 cm.
2. One angle of the triangle is 20° larger than the second angle and 3 times smaller than the third one. The sum of the first and the third angles is 120° . Find these angles.