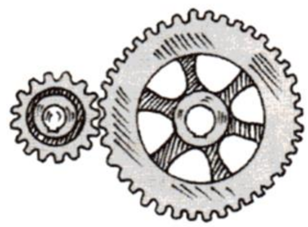
Accelerated math. Homework 10.

Problems marked with * are more difficult.

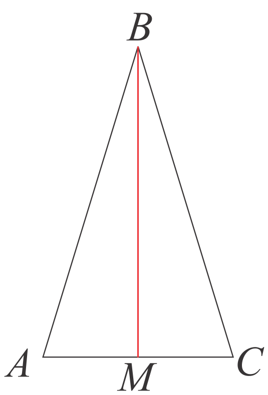
1. To solve the following problem, write an equation and solve it:
   a. Measures of three angles of a triangle in degrees, are represented by three consecutive natural numbers. Find these angles.
   b. Measures of three angles of a triangle in degrees, are represented by three consecutive even natural numbers. Find these angles.
   c. Measures of three angles of a triangle in degrees, are represented by three consecutive multiples of 3. Find these angles.
2. Draw a triangle with sides 5 cm, 6 cm, and 7 cm (use ruler and compass). Mark the midpoint of each side (use ruler), draw all three medians.
3. Draw the triangle with sides 5 cm, 6 cm, and 9 cm (use ruler and compass). Draw all three altitudes (use ruler and anything which has a right angle).
4. Open parenthesis, combine like terms and simplify the expression:

$$c-(c-d)-\left(c-\frac{d}{2}\right)-\left(c-\frac{d}{4}\right)-\left(c-\frac{d}{8}\right)-\left(c-\frac{d}{16}\right)+\frac{d}{16}$$

5. Two gears are in in clutch. One gear has 18 cogs, and another has 63. How many turns will each gear make before they both return to their original position?



6. **Theorem**. In isosceles triangle the bisector passed to the base (in isosceles triangle the base is the side different from two equal sides) is a median and an altitude as well.




Let the triangle $\Delta ABC$ be an isosceles triangle, such that $AB = BC$, and $BM$ is a bisector. We need to prove that $BM$ is a median and an altitude, which means that we need to prove that $AM = MC$ and angle $\angle BMC$ is a right angle.

$BM$ is a bisector, so $\angle ABM$ and $\angle MBC$ are equal (congruent) engles, the triangle $\Delta ABC$ is an isosceles triangle, so $AB = BC$ and the segment $MB$ is common side for triangles $\Delta ABM$ and $\Delta MBC$. Based on the Side-Angle-Side criteria, the triangles $\Delta ABM$ and $\Delta MBC$ are congruent. Therefore, $AM = MC$ and $BM$ is a median. Also we can now see that angles $\angle A$ and $\angle C$ are congruent. (Isosceles triangle has equal angles adjacent to the base).

$$\angle A + \angle B + \angle C = 180° = 2\angle A + \angle B \Rightarrow 90° = \angle A + \frac{1}{2}\angle B$$

but for the triangle ABM (as well as for MBC), $\angle A + \frac{1}{2}\angle B + \angle BMA = 180°$ , therefore $\angle BMA = 90°$ and BM is also an altitude.

Please, be ready to prove this theorem.

7. Is the number $49^4 \cdot 6^2$ divisible by 7? By 14? By 42?
8. Simplify the following expressions:

a. $aa^m(-a)^2$;

b. $c^k c(-c^2)c^{k-1}c^3$;

c. $d^n d(-d^{n+1})d^n d^2$;

d. $2x^2y^3 \cdot (-4xy^2)$;

e. $0.5a(-b)^6 \cdot 10a^2b^2$;

f. $\frac{1}{6}(-c)^3dk \cdot (-6cdk^3)$;

g. $2^4 + 2^4$;

h. $2^m + 2^m$;

i. $2^m \cdot 2^m$;

j. $3^2 + 3^2 + 3^2$;

k. $3^k + 3^k + 3^k$;

l. $3^k \cdot 3^k \cdot 3^k$;

Remember, that $(-a)^3 \neq (-a^3)$, in the first case $(-a)^3 = (-a) \cdot (-a) \cdot (-a)$, but in the second one $(-a^3) = -(a \cdot a \cdot a) = -a \cdot a \cdot a$

*Example*:

$$\frac{1}{6}(-c)^3dk \cdot (-6cdk^3) = \frac{1}{6} \cdot (-6)(-c)^3cddkk^3 = -1 \cdot \big((-c) \cdot (-c) \cdot (-c)\big)cddkk^3$$
$$= -1 \cdot (-c^3)cddkk^3 = -1 \cdot (-c^4)d^2k^3 = c^4d^2k^3$$

9. Compare (replace … with >, <, or =) if possible, if it is known that *a* and *b* are positive numbers and *x* and *y* are negative numbers:

| | | | |
|---|---|---|---|
| $0 \ \ldots \ x$ | $a \ \ldots \ 0$ | $-b \ \ldots \ 0$ | $0 \ \ldots -x$ |
| $a \ \ldots \ x$ | $y \ \ldots \ b$ | $-y \ \ldots \ x$ | $-a \ \ldots \ b$ |
| $\lvert x\rvert \ \ldots \ x$ | $-\lvert y\rvert \ \ldots \ y$ | $a \ \ldots \ \lvert a\rvert$ | $\lvert b\rvert \ \ldots \ \lvert -b\rvert$ |
| $\lvert x\rvert \ \ldots \ a$ | $\lvert x\rvert \ \ldots -x$ | $\lvert x\rvert \ \ldots -\lvert y\rvert$ | $a \ \ldots \ \lvert -b\rvert$ |

10. Positive or negative value of *m* will make the following equalities true statements?

$|m| = m$

$|m| = -m$

$-m = |-m|$

$m = |-m|$

$m = -m$

$m + |m| = 0$

$m + |m| = 2m$

$m - |m| = 2m$