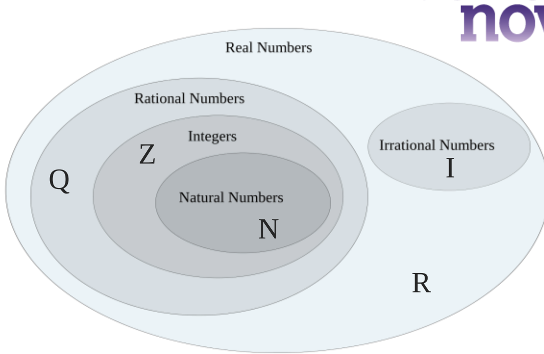
Accelerated Math. Class work 12.

Algebra.



Properties of addition and multiplication:

Let a, b, and c be real umbers.

Property	addition	multiplication
Closure	$a+b$ is a real number	ab is a real number
Commutative	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)$	$(a\cdot b)\cdot c=a\cdot(b\cdot c)$
Identity	$a+0=a, \quad 0+a=a$	$a\cdot 1=a,\ \ 1\cdot a=a$
Inverse	$a+(-a)=0$	$a\cdot\frac{1}{a}=1,\ \ a\neq 0$

The following property involves both addition and multiplication:

$$a(b+c)=ab+ac$$

Distributive property can be used to perform various arithmetic (and algebraic) transformations, mostly rewriting the given expression without parenthesis, or factor something out in the given expression.

If we need to multiply one expression by another expression – say multiply $(a+b)$ by $(c+d)$.

$$(a+b)\cdot(c+d)$$

Let's make a substitution, instead of $(a+b)$ let's use just variable $u=(a+b)$. The expression now looks easier:

$$(a+b)\cdot(c+d)=u\cdot(c+d)$$

and can be easily rewrite without parenthesis:

$u \cdot (c + d) = u \cdot c + u \cdot d$

But we remember that we have to change u back to $(a + b)$ expression.

$(a + b) \cdot (c + d) = u \cdot (c + d) = u \cdot c + u \cdot d = (a + b) \cdot c + (a + b) \cdot d$

Distributive property can be used again and we will get the following:

$(a + b) \cdot (c + d) = (a + b) \cdot c + (a + b) \cdot d = a \cdot c + b \cdot c + a \cdot d + b \cdot d$

Last expression contains product of each term of $(a + b)$ and each term of $(c + d)$.

Example:

$$\begin{aligned}(c + d) \cdot (2 + 2m + k) &= (c + d) \cdot 2 + (c + d) \cdot 2m + (c + d) \cdot k \\ &= 2c + 2d + 2cm + 2dm + ck + dk\end{aligned}$$

There are few important identities, which you should know, be able to derive:

$(a + b)^2 =$

$(a - b)^2$

$a^2 - b^2 =$

$(a + b)^3 =$

1. $(2x + 3)^2 =$
2. $(3 - 2m)^2 =$
3. $x^2 - y^2 =$
4. $(2 - m)(2 + m) =$

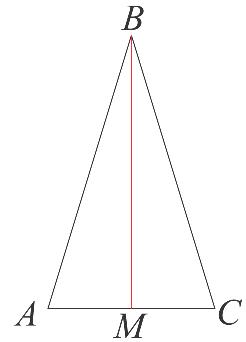
Geometry.

We proved the theorem about the angles of the isosceles triangle;

In the isosceles triangle angles at the base are equal.

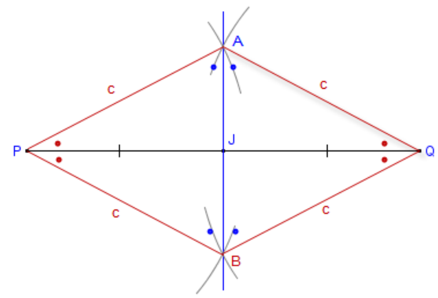
Let’s prove that if two angles of a triangle are equal, this triangle is an isosceles triangle.

In the triangle ABC $\angle BAC = \angle BCA$. Prove, that triangle ABC is an isosceles triangle. To prove this we need to prove that $|AB| = |BC|$ (or $[AB] \cong [BC]$). Let's draw the segment BM, a bisector of the angle $\angle ABC$



$\Delta ABM \cong \Delta MBC$, because angle $\angle A = \angle C, \angle ABM = \angle MBC$ (BM is a bisector), therefore $\angle BMA = \angle BMC$ and the side BM is a common side. Two triangles are congruent by the ASA test. Therefore $|AB| = |BC|$.

How to divide the segment by half with the compass and a straight edge?



	Argument	**Reason**
1	Line segments AP, AQ, PB, QB are all congruent	The four distances were all drawn with the same compass width c.
2	Triangles ΔAPQ and ΔBPQ are isosceles	Two sides are congruent (length c)
3	Angles AQJ, APJ are congruent	Base angles of isosceles triangles are congruent
4	Triangles ΔAPQ and ΔBPQ are congruent	Three sides congruent (sss). PQ is common to both.

5	Angles APJ, BPJ, AQJ, BQJ are congruent. (The four angles at P and Q with red dots)	Corresponding parts of congruent triangles are congruent
6	ΔAPB and ΔAQB are isosceles	Two sides are congruent (length c)
7	Angles QAJ, QBJ are congruent.	Base angles of isosceles triangles are congruent
8	Triangles ΔAPB and ΔAQB are congruent	Three sides congruent (sss). AB is common to both.
9	Angles PAJ, PBJ, QAJ, QBJ are congruent. (The four angles at A and B with blue dots)	Corresponding parts of congruent triangles are congruent
10	Triangles ΔAPJ, ΔBPJ, ΔAQJ, ΔBQJ are congruent	Two angles and included side (ASA)
11	The four angles at J - AJP, AJQ, BJP, BJQ are congruent	Corresponding parts of congruent triangles are congruent
12	Each of the four angles at J are 90°. Therefore AB is perpendicular to PQ	They are equal in measure and add to 360°
13	Line segments PJ and QJ are congruent. Therefore AB bisects PQ.	From (8), Corresponding parts of congruent triangles are congruent