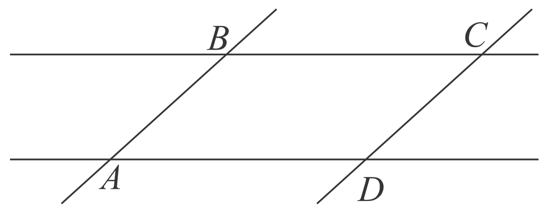
Accelerated Math. Class work 16.

Geometry.

Parallelogram is a simple (non-self-intersecting) quadrilateral with two pairs of parallel sides.

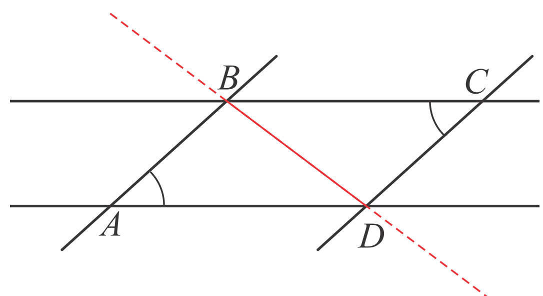


What else can we say about parallelogram? Is this enough as a definition?

Parallel sides of a parallelogram are equal. Does this statement need to be included into the definition or it can be proved based on the fact that they are parallel?

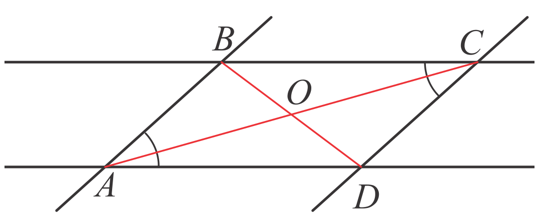
Let $ABCD$ be a parallelogram, we need to prove that $|AB| = |CD|$ and $|AD| = |BC|$.

Segment BD is a diagonal of the parallelogram. Angle $\angle ABD = \angle BDC$ as alternate interior angles (we can prove it based on the 5th Euclid postulate), as well as angles $\angle BDA = \angle DBC$. BD is a common side for the triangles. Therefore, triangles ABD and BDC are congruent, based on the ASA criteria and have equal corresponding sides. Conclusion: $|AB| = |CD|$ and $|AD| = |BC|$.



Opposite angles of a parallelogram are equal, and adjacent angles are supplementary.

Diagonals of a parallelogram intersect at midpoint of both segments. Prove it. Also prove the converse theorem – if two diagonals of a quadrilateral intersect at midpoint of both, then this quadrilateral is a parallelogram.



Algebra.

1. Instead of M and N put the right expressions to get a true statement.
   a. $(a+b+c)+(M-N+c)=4a-2b+2c$
   b. $(7x-N)-(M+2y)=3x-3y$
   c. $(M+N)-(2a-b)+(a-4b)=5a+7b$
   d. $(a-M)-(N+7b)-(2a+b)=-5a-10b$
   e. $2\cdot(M-b)=14a-2b$
   f. $M(2a+3b)=-6a-9b$
   g. $N\cdot(2x-M)=12x^2-18xy$
   h. $3a\cdot(N+M)=15abc-3ac^2$
2. Knowing that $A=a+b$, $B=3a+3b$, and $C=a-7b$ find:
   a. $A+B+C$
   b. $A+B-C$
   c. $A-B-C$
   d. $-B-A-C$
3. Simplify the following expressions:
   a. $a-(b-(a+b)-a)$
   b. $a-\Big(a-\big(a-(a-b)\big)\Big)$
   c. $a-\Big(b-\big(a-b-(a-b)\big)\Big)$
   d. $b-\Big(a-\Big(a-\big(a-(a+b)\big)\Big)\Big)$
4. Prove that

$$(n+1)!-n\cdot n!=n!$$

   (We use ! as the sign for the term "factorial", $n!=n\cdot(n-1)(n-2)\cdot\ldots\cdot3\cdot2\cdot1$)

5.

$(a+1)(a+1)(a+1);$	$(x-1)(x-1)(x-1);$
$(a+b)(a-b)(a+b);$	$(m-n)(m-n)(m+n);$
$(a+b+c)(a+1);$	$(a-b-c)(a-1);$
$(x+1)(x^2-x+1);$	$(x-1)(x^2+x+1);$
$(x^3+2x-3)(2-3x);$	$(5m^2-3mn+n^2)(2n-m^2);$
$(a+b+c)(a+b-c);$	$(a-b+c)(a-b-c).$
$-(a+b)(a+b);$	$-(x-y)(x+y);$
$-(x-y)(x-y);$	$-(2m-n)(n-3m);$
$-(5a-2b)(3b+2a);$	$-7(x+8y)(y-3x).$