Accelerated Math. Class work 18.

Algebra.

Positive rational number is a number which can be represented as a ratio of two natural numbers:

$$a = \frac{p}{q}; \qquad p, q \ \in N$$

As we know such number is also called a fraction, p in this fraction is a nominator and q is a denominator. Any natural number can be represented as a fraction with denominator 1:

$$b = \frac{b}{1}; \ \ b \in N$$

Basic property of fraction: nominator and denominator of the fraction can be multiply by any non-zero number n, resulting the same fraction:

$$a = \frac{p}{q} = \frac{p \cdot n}{q \cdot n}$$

In the case that numbers p and q do not have common prime factors, the fraction $\frac{p}{q}$ is irreducible fraction. If $p < q$, the fraction is called “proper fraction”, if $p > q$, the fraction is called “improper fraction”.

If the denominator of fraction is a power of 10, this fraction can be represented as a finite decimal, for example,

$\frac{37}{100} = \frac{37}{10^2} = 0.37,$ $\qquad \frac{3}{10} = \frac{3}{10^1} = 0.3,$ $\qquad \frac{12437}{1000} = \frac{12437}{10^3} = 12{,}437$

$$10^n = (2 \cdot 5)^n = 2^n \cdot 5^n$$

$$\frac{2}{5} = \frac{2}{5^1} = \frac{2 \cdot 2^1}{5^1 \cdot 2^1} = \frac{4}{10} = 0.4$$

Therefore, any fraction, which denominator is represented by $2^n \cdot 5^m$ can be written in a form of finite decimal.

Also, any finite decimal can be represented as a fraction with denominator 10^n.

$0.375 = \frac{375}{1000} = \frac{3}{8} = \frac{3}{2^3};$ $\qquad 0.065 = \frac{65}{1000} = \frac{13 \cdot 5}{5^3 2^3} = \frac{13}{5^2 2^3};$

$6.72 = \frac{672}{100} = \frac{168}{25} = \frac{168}{5^2};$ $\qquad 0.034 = \frac{34}{1000} = \frac{17 \cdot 2}{5^3 2^3} = \frac{17}{5^3 2^2};$

In other words, if the finite decimal is represented as an irreducible fraction, the denominator of this fraction will not have other factors besides 5^m and 2^n. Converse statement is also true: if the irreducible fraction has denominator which only contains 5^m and 2^n than the fraction can be written as a finite decimal. (Irrdeuceble fraction can be represented as a finite decimal if and only if it has determinator conteining only 5^m and 2^n as factors.)

If the denominator of the irreducible fraction has a factor different from 2 and 5, the fraction cannot be represented as a finite decimal. If we try to use the long division process we will get an infinite decimal.

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         . .
         0.0675
    74 ) 5.00000
         4.44
           560
           518
            420
            370
             500
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At each step during this division we have a remainder. They are the numbers from 1 to 73. At some point during this division we will see the remainder which occurred before. Process will start to repeat itself.

How we can represent the periodic decimal as a fraction?

Let's take a look on a few examples: $0.\overline{8}$, $2.35\overline{7}$, $0.\overline{0108}$.

1. $0.\overline{8}$. $x = 0.\overline{8}$ $10x = 8.\overline{8}$ $10x - x = 8.\overline{8} - 0.\overline{8}$ $= 8$ $9x = 8$ $x = \frac{8}{9}$	2. $2.35\overline{7}$ $x = 2.35\overline{7}$ $100x = 235.\overline{7}$ $1000x = 2357.\overline{7}$ $1000x - 100x$ $= 2357.\overline{7} - 235.\overline{7}$ $= 2122$ $x = \frac{2122}{900}$	3. $0.\overline{0108}$ $x = 0.\overline{0108}$ $10000x = 108.\overline{0108}$ $10000x - x = 108$ $x = \frac{108}{9999}$

Now consider $2.4\overline{0}$ and $2.3\overline{9}$

$x = 2.4\overline{0}$

$100x - 10x = 240 - 24$

$10x = 24.\overline{0}$

$100x = 240.\overline{0}$

$$x = \frac{240 - 24}{90} = \frac{216}{90} = 2.4$$

$x = 2.3\overline{9}$

$10x = 23.\overline{9}$

$100x = 239.\overline{9}$

$100x - 10x = 239 - 23$

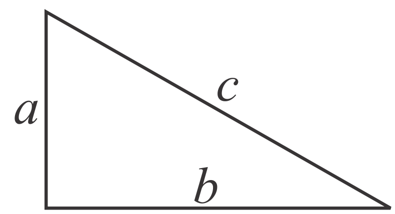
$$x = \frac{239 - 23}{90} = \frac{216}{90} = 2.4$$

Exercises.

Write as a fraction:

$0.\overline{3}$, $0.\overline{7}$, $0.1\overline{2}$, $1.12\overline{3}$, $7.5\overline{4}$, $0.\overline{12}$, $1.0\overline{12}$.

Geometry.

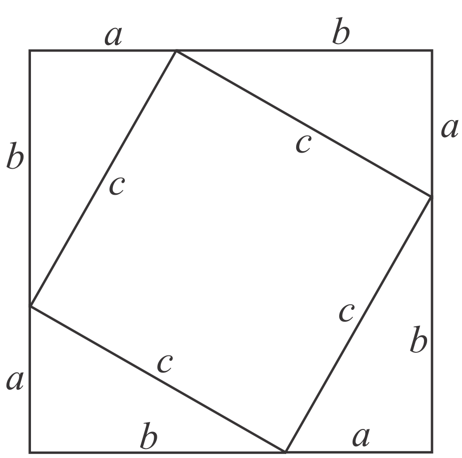


Pythagorean theorem.

In a right triangle the side opposite the right angle is called the hypotenuse (side c in the figure). The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).

Pythagorean theorem is states that the square of the hypotenuse is equal to the sum of square of legs. $c^2 = a^2 + b^2$

There are numneros proofs if this theorem, but let's see one.



4 identical right triangles are arranged as shown on the picture. He area of the big square is $S = (a + b) \cdot (a + b) = (a + b)^2$, the are of the small square is $s = c^2$. The area of 4 triangles is $4 \cdot \frac{1}{2}ab = 2ab$. But also cab be represented as $S - s = 2ab$

$$2ab = (a + b) \cdot (a + b) - c^2 = a^2 + 2ab + b^2 - c^2$$

$$\Rightarrow \qquad a^2 + b^2 = c^2$$