School Nova
Math Battle
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1. Circle!:
   Points A, B, C, and D lie on a circle, in that order, such that $AB = 3$, $BC = 5$, $CD = 6$, and $DA = 4$. The diagonals AC and BD intersect at point P. Compute $\frac{AC}{AP}$.
   HMMT 2017 February; Geometry, Problem 1

2. Algebra Trick or Tricky Algebra?:
   Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(x)f(y) = f(x - y)$ for all x, y. Find all possible values of $f(2017)$.
   HMMT 2017 February; Algebra and Number theory, Problem 3

3. Rooks on a Chess Cube:
   In chess, rooks are pieces that may move to any square in the same row or column that they are currently in. If a rook is able to move onto a piece, it is said to be attacking that piece. Suppose you have a box full of rooks: what's the maximum number of rooks you can place on the board (8x8 chessboard) so that none of them are attacking each other?
   Now consider you have an 8x8x8 chess cube where pieces occupy a single little cube of the grid. Rooks will be allowed to move parallel to the sides of the cube: they may move to any cube in the same row, column, or tower (straight line in the third dimension) that they are currently in.
   What's the maximum number of rooks you can place on the 8x8x8 chess cube so that none of them are attacking each other?

4. Paper Folding:

   Is it possible to fold a square origami paper in a way that you end up with a section whose area is exactly 1/3 the area of the paper? For which n is it possible to fold over a section whose area is 1/n that of the paper?

   Origami rules:

   The four sides of the square are 'lines' the four corners are 'points', and these are the only lines and points you start with. You may create a new point at the intersection of any two lines, and you may create new lines as: (i) the line through two points, (ii) the midline between two lines (the midline is the line of points equidistant from the two lines), (iii) the perpendicular to a line that passes through a point, (iv) the midline between two points, (v) and, lastly, you may reflect any point or line through an existing line (ie you may fold the paper over a line and re-fold all creases to get their mirror images). These are the only allowed moves.

   Note: these rules are named after mathematicians Huzita and Hatori, called the Huzita-Hatori axioms (except the last one), and include three more axioms which are uninteresting to this problem but useful in other adventures.

5. Black and White Stickers:

   Ten students are in a classroom, wanting to leave, and the teacher decides to dismiss students according to their success in the following game: the teacher will put a black or a white sticker on each student's forehead (one per student, the color randomly chosen), so that the students can see each others' stickers but not their own. The teacher will then ask the students, one by one, to guess the color of their sticker. If they get it right, they may leave; if not, they must stay for the next class (which is very boring). During the game, starting from when the teacher places the first sticker, the students may not communicate at all, except that they may hear each others' guesses. The students are given five minutes to talk to each other before the game begins. They want to devise a strategy to save as many of them as possible. What's the best strategy? (Note: if anyone cheats, all students must stay for the next class.)