MATH CLUB: COMBINATORICS

DEC 3, 2017

Recall the basic rules of combinatorics:
The number of ways to choose k items out of n if the order in which they are chosen matters is

$$_nP_k = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}.$$

The number of ways to choose k items out of n if the order in which they are chosen doesn't matter is

$$_nC_k = \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}.$$

For example, the number of sequences of length n consisting of k zeros and $n-k$ ones is $_nC_k$ (this is equivalent to choosing k positions where we put zeros).

1. How many ways there are to divide 10 students into two teams of 5? (Teams do not have names)

2. How many ways there are for 15 students to take seats in a classroom with 30 chairs?

3. How many ways there are to group $2n$ people into n pairs?

4. How many different "words" can be formed by permuting letters of the word "Mississippi"? (by "word", we mean any sequence of letters, not necessarily meaningful").

5. How many different necklaces can one make using 1 green, 5 red, and 7 blue beads? using 2 green, 5 red, and 7 blue beads?

6. How many ways there are to arrange 12 books on 2 bookshelves (top and bottom one)? The order on each bookshelf matters; there are no restrictions on how many of the 12 books are on top shelf.

7. How many different monomials in 3 variables x, y, z of total degree n are there? in 4 variables?

8. (a) In a $n \times n$ square, how many paths are there from the bottom left to the top right corner, if we are only allowed to move up and to the right, never down or to the left?

*(b) What if we additionally require that the path can never go below the diagonal?

9. (a) n real numbers are placed around a circle. It is known that the sum of them all is nonnegative. Show that one choose a starting point on the circle so that if we start travelling counterclockwise from this point and denote the numbers in the roder we meet them by $a_1, a_2, \dots a_n$, then

$$\begin{aligned} &a_1 \geq 0 \\ &a_1 + a_2 \geq 0 \\ &a_1 + a_2 + a_3 \geq 0 \\ &\dots \end{aligned}$$

(b) Along a loop of desert road, there are n fuel storage stations. It is known that the total amount of fuel in them is sufficient for a car to go around the whole loop. Prove that it is possible to find a starting position so that the car starting from this point and travelling counterclockwise can complete the loop, using the fuel it finds along the way. (The car starts with no fuel.)