

## MATH CLUB: MATH BATTLE

DEC 17, 2017

1. A teacher writes 10 natural numbers on the board. Show that it is always possible to choose some of them and put signs + and - between them so that the resulting expression is a multiple of 1001.
2. A settlement consists of 9 blocks, forming a  $3 \times 3$  square, each block itself a square with side  $a$ . To deliver mail, the postman has to walk each of the streets (including the streets forming the outer boundary) at least once. What is the smallest distance he has to walk?
3. One hundred white stones are placed in a circle. We are given an integer  $k$  ( $1 \leq k \leq 50$ ) and are allowed to perform the following operation: choose  $k$  stones in a row, with 1st and last of them white, and color the 1st and last black.  
For which  $k$ , after repeating this operation several times, we can color all stones black?
4. Find all integer  $n > 2$  such that the polynomial  $x^n + x + 1$  is divisible by the polynomial  $x^2 + x + 1$ ?