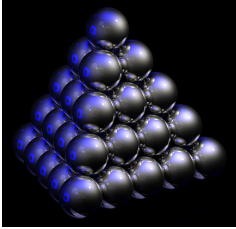


## MATH CLUB: SEQUENCES AND FINITE DIFFERENCES

APRIL 8, 2018

1. Can you find a sequence  $a_n$  such that  $a_n - a_{n-1} = n$ ? How many such sequences are there?  
Can you do the same if instead we require that  $a_n - a_{n-1} = n^2$ ?
2. Can you continue each of the following sequences?
  - (a) 3, 5, 7, 9,
  - (b) 1, 3, 6, 10, 15,
  - (c) -1, 2, 9, 22, 43, 74, 117Can you also write a formula for  $n$ -th term for each of these sequences?
3. Consider a pyramid of balls, such as the one below:



How many balls does it contain if the number of layers is equal to  $n$ ?

4. Each side of the triangle is divided into  $n$  equal parts. These points are connected by lines, parallel to the sides of the triangle (thus, we get 3 families of lines: parallel to side  $AB$ , parallel to side  $BC$ , parallel to side  $AC$ ).  
Into how many triangles do these lines divide the original triangle?
5. Each edge of a tetrahedron is divided into  $n$  equal parts. These points are connected by planes, parallel to the faces of the tetrahedron (thus, we get 4 families of planes — one family for each face of the tetrahedron).  
Into how many tetrahedrons do these planes divide the original tetrahedron?
6. If we have  $n$  lines in a plane such that no two lines are parallel, and no three lines intersect at a common point, into how many regions do these lines divide the plane?
7. A regular tetrahedron is rolled on a plane without slipping. Is it possible to roll it so that it comes back to the original position in the plane, but resting on a different face?  
[Hint: color the plane!]