

MATH 8-10
THE FINAL BATTLE!
MAY 20, 2018

1. Given a circle and a point P inside it, construct a chord AB through P such that $AP - BP = 1$ cm.
2. 11 people sit at a round table. On the table one places a rotating tray. Can you put on this tray 11 name cards, with the names of these 11 people, so that no matter how one rotates the tray, at least one of the cards matches the name of the person?
3. A 10×10 table is filled with whole numbers, some of them negative. You are allowed to perform the following operations
 - Change sign of all the numbers in some row of the table
 - Change sign of all the numbers in some column of the table

Is it true that by using these operations, you can make the sum of numbers in each row and in each column non-negative? [Hint: is there an invariant? a semi-invariant, i.e. something that always increases?]

4. Igor and Alex are playing the following game. They have a staircase with 1001 steps; on some of these steps, there are stones (no more than one stone on each step). At his turn, Igor can take any stone and move it to the nearest free step above it. After that Alex can take any stone such that the step immediately below it is empty and roll this stone one step down.

Initially there are 500 stones, occupying steps 1–500. Igor goes first; his goal is to get a stone to step 1001. Alex's goal is prevent Igor from doing this.

Is there a winning strategy for one of them? if so, what is this strategy?

5. A plot of land has the the shape of a square with side 10. It is known that this plot is crossed by a buried water pipe, going in a straight line. A plumber wants to find the pipe, by digging narrow trenches until one of them hits the pipe.

What is the shortest total length of trenches he has to dig to guarantee that he will find the pipe? One option is digging the trenches along two diagonals of the square. Can you do better than that?