Math 4. Class work 3.

Algebra

Addition | **Multiplication**

Commutative property

- Addition: $a+b=b+a$
- Multiplication: $a\times b=b\times a$

Associative property

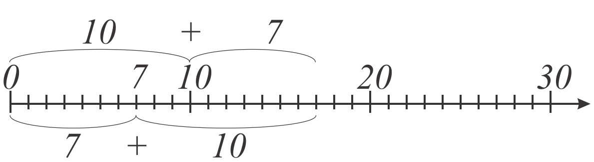
- Addition: $a+(b+c)=(a+b)+c$
- Multiplication: $a\times(b\times c)=(a\times b)\times c$

Distributive property

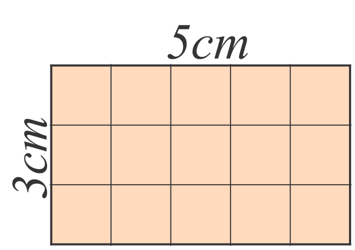
$$a\times(b+c)=a\times b + a\times c$$

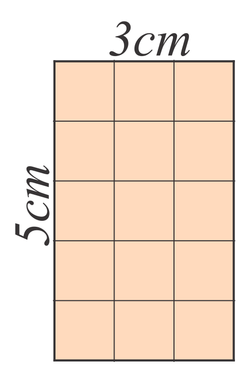
$$a\times(b - c)=a\times b - a\times c$$

Commutative and associative properties are intuitively easy to understand.



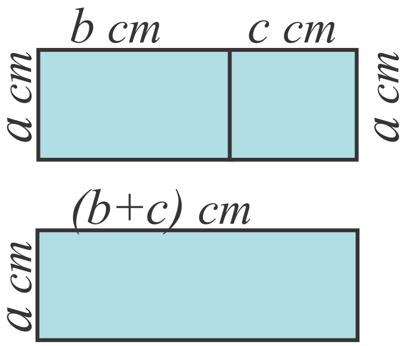
$$S = 3cm \times 5cm = 5cm \times 3cm$$
$$= 15cm^2$$





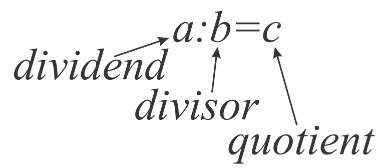
Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes? Is there any difference between these 2 expressions:

$$5 \times (10 + 8) \quad or \quad 5 \times 10 + 5 \times 8$$



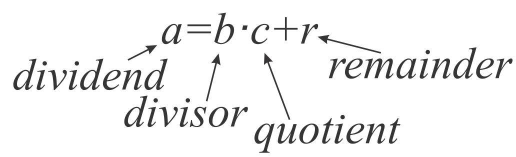
Let's consider this in another example: The combined area of these 2 rectangles is $S = a \times b + a \times c$ but the rectangle with one side a cm and the other $(b+c)$ cm will have exactly the same area.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.



If a and n are natural numbers, the result of division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$



Where r is a remainder of division $a \div b$. If r is 0, then we can tell that a is divisible by b.

Divisibility rules.

Can we predict whether a given number is divisible by 2, 3, 4, and so on? There exist the following divisibility rules.

1. ***A number is divisible by 2 if and only if its last digit is even or 0.***
   Is this statement is true for any number? Let's check:

$$124: 2 = 62, \qquad 1345672: 2 = 672836$$

Can we prove that this statement is true based on several checks? Some statements can just be checked. For example, the statement “This girl’s name is Sophia” can be easily checked, we just need to ask the girl about her name.
Can we as easily check the statement “All girls’ names is Sophia” ? What do we have to do in order to tell whether this statement is true or false? To confirm that all girls in the world are called Sophia I need to ask all of them. To find out that this is not true I just need to find only one girl with another name.
In our case of divisibility question I need to find the way to prove that is true for any natural number, because I just can’t check all of them.
Proof of the divisibility by 2 rule:
Any natural number can be written as a sum:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + k$$

If k is an even number, it also can be represented as a product of 2 and something else. Then the number can be written as:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + 2 \times p$$

(p can be 0, 1, 2, 3, and 4. Do you know why?). Distributive property is allowing us to represent this expression as a product:

$$\ldots + 1000 \times n + 100 \times m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + k$$
$$= 2 \times (\ldots + 500 \times n + 50 \times m + 5 \times l + p)$$

Now we can see that the number is divisible by 2 if its last digit is even or 0.

2. ***A number is divisible by 3 if and only if sum of its digits is divisible by 3.***

3. ***A number is divisible by 4 if and only if the number formed by the last 2 digits is divisible by4.***

4. ***A number is divisible by 5 if and only if its last digit is 5 or 0.***

5. ***A number is divisible by 6 if it is divisible by 2 and 3 at the same time, so it will be divisible by 6 if an only if its last digit is even or 0 and the sum of its digits is divisible by 3.***

Exercises. Problems with * are more difficult.

1.
   a. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating and dividing?

b. Find all natural numbers such that upon division by 7 they give equal quotient and remainder.

2. Factorize (represent as a product of 2 or more multipliers) the following expressions:

   Example: $3 \times 5 + 3 \times 7 = 3 \times (5 + 7)$

   a. $2 \times 3 + 2 \times 5 =$
   b. $3x + 3y =$
   c. $5a + 5b + 5c =$
   d. $ab + ac =$
   e. $ma - mb$
   f. $ds + dk - dl$

3. If we want to divide m by 15, what numbers we can get as a remainder?

4. Andrew prepares for an ironman competition. For that he swims for 37 minutes every day during 256 days and also he runs for 63 minutes every day during 256 days. How many minutes does he spend doing sports?

5. Rewrite the following expression without parenthesis:

   a. $2 \times (a + b) =$
   b. $a(x + y) =$
   c. $(a + 2) \times 5 =$

6. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number
   Can you explain why?

7.
   a. Will the following numbers be divisible by 2:

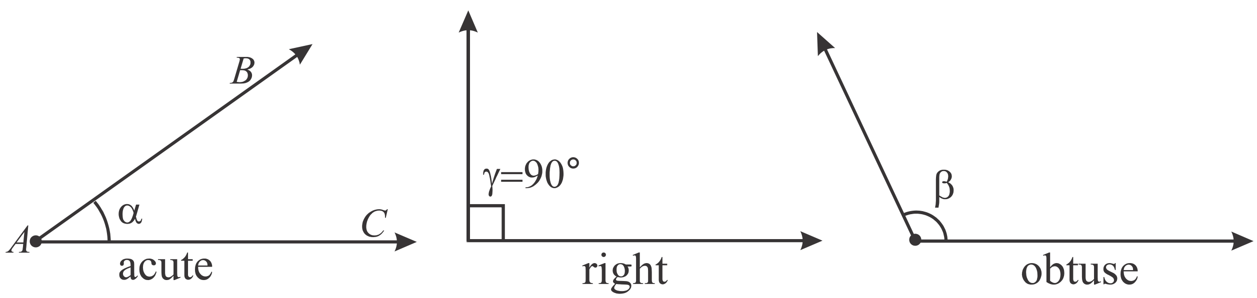
      123457, 1029384756, 43567219874563157830

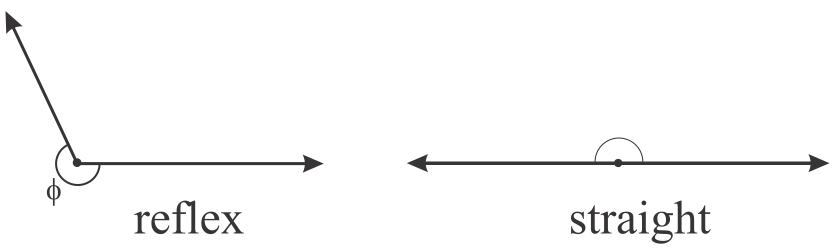
   b. by 3
      1347, 45632, 56379842659
   c. by 5:
      5635, 78530, 657932, 45879515

Geometry.

An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

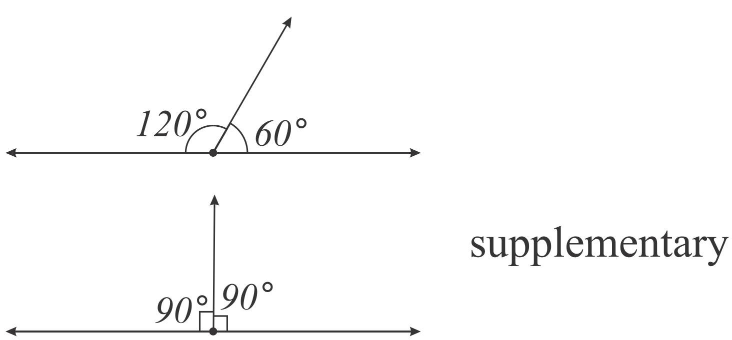
Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: ∠ABC, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:

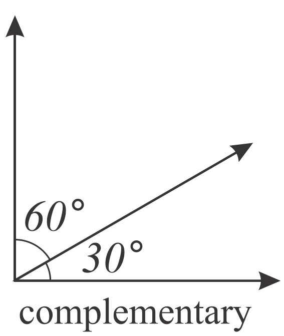




Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

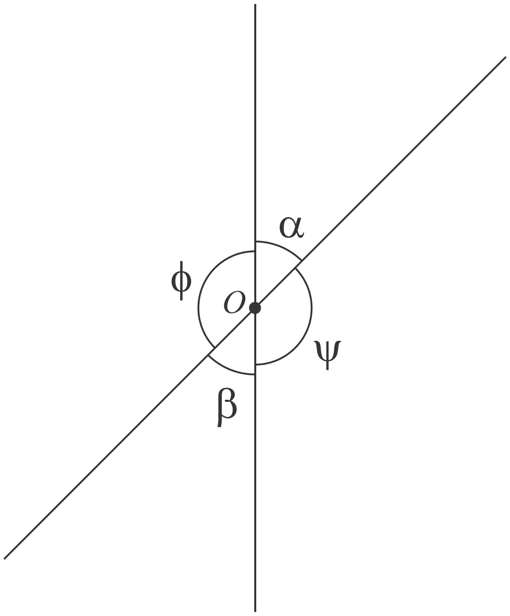
Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.





An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem:

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

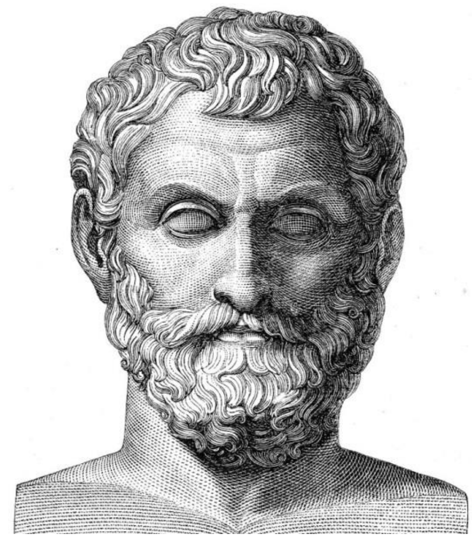
Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$, therefore we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Exercises.

1. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
2. * 3 lines intersect at 1 point and form 6 angles. One is 44°, another is 38°. Can you find all other angles?
3. *Right angle is divided into 3 angles by 2 rays. One of this angles by 20° more than the other and by 20° less the third one. What are the measures of these 3 angles?
4. On the picture below ∠BOD = 152°, ∠COD = 55°, angle ∠AOD is a straight angle. Find the measures of all other angles on the picture.