

Algebra.

Multiplication and division of fractions.

Peter and 2 of his friends want to divide 2 chocolate bars evenly:

$$2 \div 3 = \frac{2}{3} = 2 \times \frac{1}{3}$$

You can see that the division by 3 (to find $\frac{1}{3}$ of 2 chocolate bars) is the same as multiplication by $\frac{1}{3}$. Also Peter has 18 candies. He wants to give to his friends $\frac{2}{3}$ of his candies. To find how many candies Peter should give to his friends he has to divide the number of candies by 3 and multiply by 2:

$$18 \div 3 \times 2 = 18 \times 2 \div 3 = \frac{18}{3} \times 2 = \frac{18 \times 2}{3} = 18 \times \frac{2}{3} = \frac{2}{3} \times 18$$

Fractions.

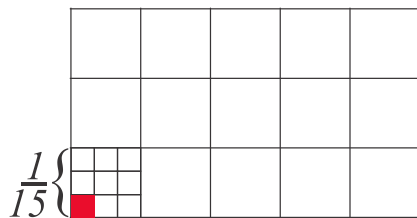
Multiplication of fraction.

What does it mean to multiply one fraction by another?

If we want to multiply a whole number by a fraction, it means we are dividing this number by the denominator of the fraction and are multiplying by the numerator and, as a result, we will get fraction part of our whole number.

$$\frac{2}{3} \text{ of } 15 \text{ is } 15 \div 3 \times 2 = 10 = \frac{2}{3} \times 15$$

Now we want to find $\frac{1}{9}$ out of $\frac{1}{15}$.



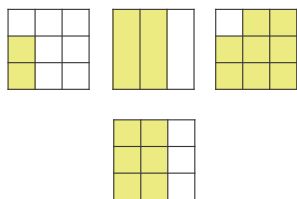
$$\frac{1}{9} \times \frac{1}{15} = \frac{1}{15} \div 9 = \frac{1}{15 \times 9} = \frac{1}{135}$$

$$\frac{2}{9} \times \frac{1}{15} = \frac{1}{15} \div 9 \times 2 = \frac{1 \times 2}{15 \times 9} = \frac{2}{135}$$

Let's try to add $\frac{2}{9}$ and $\frac{2}{3}$. What should we do? Why do we need to bring both fractions to the same denominator? We can add together only similar objects: apples to apples and oranges to oranges. Are two fractions $\frac{2}{9}$ and $\frac{2}{3}$ similar objects?

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}, \quad \frac{2}{9} = \frac{1}{9} + \frac{1}{9}$$

How we can add together



$$\frac{2}{9} + \frac{2}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{3} + \frac{1}{3}$$

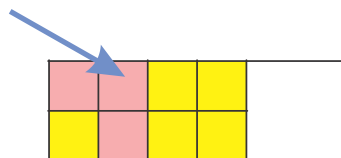
To be able to add two fractions we have to be sure that they have the same denominator. Each $\frac{1}{3}$ is exactly the same as $\frac{3}{9}$ and $\frac{2}{3} = \frac{6}{9}$

$$\frac{2}{3} \times 1 = \frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3}$$

If we multiply both numerator and denominator by the same number the fraction will not change. To bring 2 fractions to the same denominators we have to multiply the numerators and the denominators of both fractions by two different numbers to get a common multiple as the denominator for both fractions. There are many common multiples of 2 numbers. Of course, one of them is their product, but is not always the simplest one. Usually, it is convenient to find LCM of these 2 (or, sometimes more than 2) numbers.

More of fraction multiplication:

$$\frac{3}{8} \times \frac{2}{3} = \frac{2}{3} \div 8 \times 3 = \frac{2 \times 3}{8 \times 3} = \frac{2}{8} = \frac{1}{4}$$



What does it mean that $\frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$? It really means that $\frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$. It is easy to see that

$$\frac{1}{4} \times \frac{8}{3} = \frac{1 \times 8}{4 \times 3} = \frac{2}{3}$$

And $\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$. So

$$\frac{1}{4} \times \frac{3}{2} = \frac{1 \times 3}{4 \times 2} = \frac{3}{8}$$

Let's try to look at this problem differently. If we are saying that $10 \div 2 = 5$ it means that 2 can fit into 10 five times (and $10 \div 5 = 2$ means that 5 can fit into 10 two times.)

Expression $\frac{3}{10} \div \frac{3}{5} = \frac{1}{2}$ means that $\frac{3}{5}$ can fit into $\frac{3}{10}$ only $\frac{1}{2}$ times.



We can see that to multiply 2 fractions one should multiply numerators and denominators. To divide one fraction by another we have to multiply first fraction by the fraction reciprocal to the second fraction. Two fractions are called reciprocal (or inverse) if their product is equal to 1.

$$\frac{3}{5} \times \frac{5}{3} = 1, \quad \frac{a}{b} \times \frac{b}{a} = 1$$

1. Compute:

$$\frac{2}{3} - \frac{1}{4} =$$

$$\frac{7}{15} - \frac{1}{5} =$$

$$\frac{5}{12} + \frac{4}{15} =$$

2. Simplify the following fractions:

$$\frac{22}{66}, \frac{125}{75}, \frac{75}{100}, \frac{24}{360}, \frac{125}{1000}, \frac{100}{250}, \frac{198}{126},$$

$$\frac{2 \cdot 3}{4 \cdot 5}, \frac{2 \cdot 3}{7 \cdot 2}, \frac{5 \cdot 4}{4 \cdot 9}, \frac{7 \cdot 5}{2 \cdot 7}$$

3. Compute

$$\frac{9 \cdot 4 \cdot 5}{20 \cdot 8 \cdot 15} =$$

$$\frac{2}{3} \cdot \frac{6}{7} \cdot \frac{1}{10} =$$

$$\frac{2}{9} \cdot \frac{3}{7} \div \frac{5}{12} =$$

$$\frac{1}{3} \cdot 90 \cdot \frac{1}{5} =$$

$$\frac{9}{10} \div \frac{10}{11} \cdot \frac{100}{21} =$$

$$36 \cdot \frac{1}{12} \cdot \frac{1}{3} =$$

4. $\frac{12 \times 5 + 12 \times 9}{12 \times 21} =$

$$\frac{14 \times 5 + 14 \times 2}{28} =$$

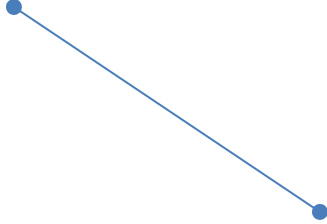
$$\frac{8 \times 8 + 8 \times 7}{8 \times 5} =$$

$$\frac{19 \times 8 + 19 \times 6}{38} =$$

5. There are three short stories in a book. Paulina read the first story in $\frac{1}{3}$ of one hour. She spent $\frac{1}{6}$ of an hour more reading the second story than reading the first one. The third story she read in $\frac{7}{12}$ of an hour less than the two previous stories together. How much time did it take her to read this book?

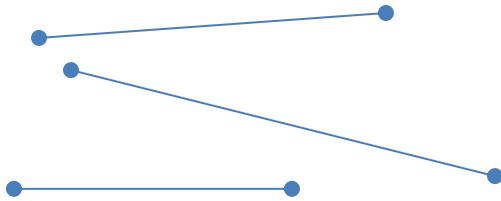
Geometry.

Draw the segment, equal to the given segment using a compass and a ruler (straight edge):



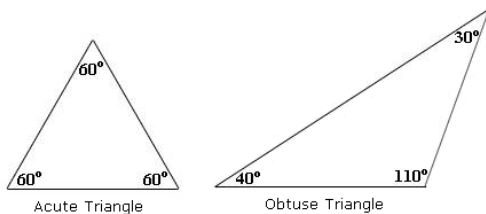
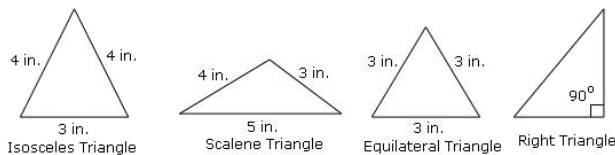
1. Mark any point on a plane.
2. Open the compass on length of the given segment.
3. Draw the part (or the whole circle).
4. Connect the marked point and any point of the circle.

How to construct the triangle with sides equal to the given segments:



Triangle.

Triangle is a closed figure consisting of three line segments linked end-to-end.



Acute triangle has all acute angles. Obtuse triangle has an obtuse angle. Can a triangle have more than one obtuse angle? Isosceles triangle has two equal sides. Equilateral triangle has all three sides equal. Right triangle has a right angle.