Math 4a. Classwork 21.

Algebra.

Let's take a look at the statement "New York City is the capital of the United States". We, definitely, can say is it True or False. Of cause it's not true, we all know that the capital of the US is Washington, DC. So we can say "it is not true, that New York City is the capital of the US", or, in a little more usual language, "New York City isn't the capital of the US". The last statement is a true statement.

"New York City is the capital of the United States" False

"New York City isn't the capital of the US" (negation) True

If the statemen is True , its negated version has to be False and vice versa. They can't be both True or both False. This rule of the math logic is one of the oldest and is called "The law of the excluded middle".

Let's try to construct negation of the several statements.

a. Number 111111111 is a prime number.
b. There is nothing on the table.
c. 0.5 and $\frac{1}{2}$ are not equale.
d. The area of a rectangle is equal to the product of its length and width.
e. Sum $18 \cdot 946 + 456$ is divisible by 9.
f. $45784 > 45784$
g. $345 < 12345$
h. All birds can flight.
i. All marine animals are fish.
j. Some students like math.
k. All natural numbers are divisible by 3.
l. Penguins live on the North Pole.
m. Polar bears live on the South Pole.

Using the law of the excluded middle prove, that the negation of statement was made incorrectly.

	Statement	Negation
1	All cats are gray.	All cats are not gray
2	Some berries are sweet.	Some berries are not sweet.
3	There are 30 days in some months.	There are no 30 days in some monthes.
4	Sometimes it is raining in the morning.	It is never raining in the morning.

We can create more complex statements, for example

I like math and physics. (I like math and I like physics.)

In which case this statement will be a true statement?

1. I like math, but I don't like physics.
2. I like physics, but I don't like math.
3. I like both, math and physics.

To negate the statement, we have to make the negation which is false. What do you think such statement will look like?

Exercise:

Mother told Mary, that she can ply videogames if she will do her homework, also will do her room, and will do dishes after diner. Will Mary play videogames if she

1. Did her room?
2. Did her room and dishes?
3. Did her homework?
4. Did her homework and dishes?
5. Did all three assignments?

On the other day mother told Mary that she will play videogames if she will do her homework or will do dishes. Will Mary play the videogames if she

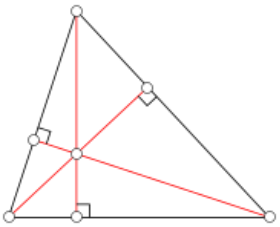
1. Did her homework?
2. Did the dishes?
3. Did both assignment?

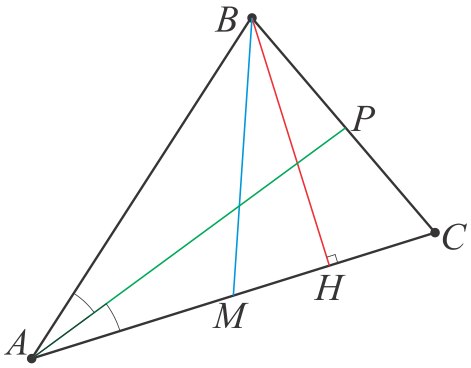
In real life your parents will probably say “You will play if you either do your homework or will do the dishes.” The phrase “either…or” in language usually means one out of two possibilities, not both, but I don’t think your parents will complain if you will do homework and dishes. In math logic the statement “Either A or B” is true if A is true, B is true, both A and B are true”.

Geometry.

Special segments of a triangle.

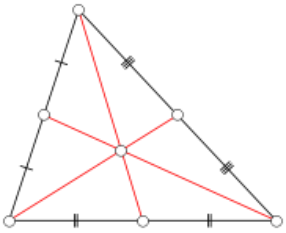
From each vertices of a tringle to the opposite side 3 special segment can be constructed.



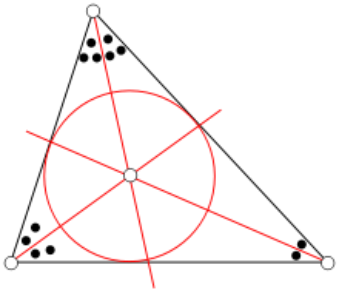


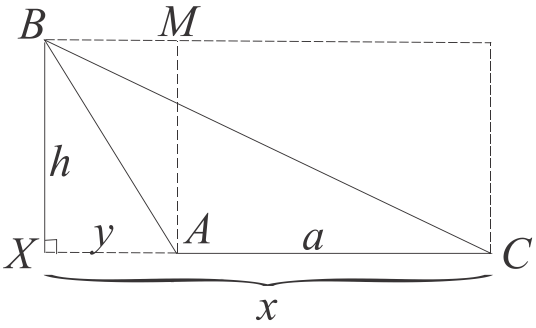
An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.

An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.



A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.





For an obtuse triangle, for one out of the three heights, it is not so obvious.

$$S_{\Delta XBC} = \frac{1}{2}h \times x, \qquad S_{\Delta XBA} = \frac{1}{2}h \times y$$

$$S_{\Delta ABC} = S_{\Delta XBC} - S_{\Delta XBA} = \frac{1}{2}h \times x - \frac{1}{2}h \times y$$
$$= \frac{1}{2}h \times (x - y) = \frac{1}{2}h \times a$$