

**Algebra.****1. Equalities: equations and identities****Inequalities.**

Is there any difference between two following equalities?

$$a(b + c) = ab + ac$$

$$a + 2 = 6$$

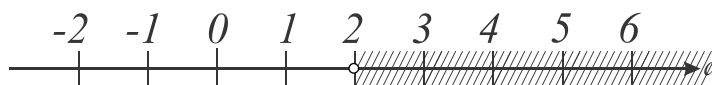
Letters  $a$ ,  $b$ , and  $c$  in both these expressions are called *variables*, we can put any number (whole or fraction) into it. In the first case the equality is still a true expression for any  $a$ ,  $b$ , and  $c$ , this is a distributive property of addition.

The second expression is a true expression for only one value of  $a = 4$  and we call this kind of expressions “an equation”. An equation is the problem of finding values of some variables, called *unknowns*, for which the specified equality is true. We have to solve the equation to find the value of an unknown variable.

There are another kind of mathematical statements – inequalities.

What will be the answer for the following question :

Which  $a$  can satisfy the statement:  $a > 2$ ?

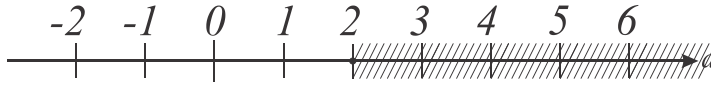


As we can see all  $a$  which lie on the right side of number 2 will satisfy the expression  $a > 2$ . What about number 2 itself? Number 2 does not satisfy our expression. How we can write the answer for  $a > 2$ ?

The best way to write the answer in terms of set theory:  $a \in ]2, \infty)$  ( $a \in (2, \infty)$ ), or the answer is set of points of number line located on the right side of number 2.

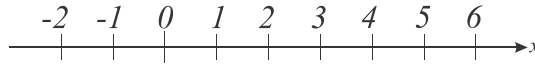
Now let's solve the inequality  $a \geq 2$ .

In this case number 2 itself also belongs to the set of numbers satisfying to our inequality and the answer will be  $a \in [2, \infty)$



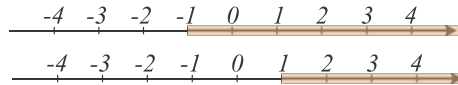
Can you find all  $x$  satisfying two following inequalities at the same time:

$x \geq -1$  and  $x < 5$ ? Write the set  $X$  containing all whole numbers satisfying these two inequalities at the same time? The answer is :



We can add any number to both part of the inequality, the sign ( $<$  or  $>$ ) will not change:

$$x > -1$$



$$x + 2 > -1 + 2 \Rightarrow x + 2 > 1$$

$$y - 3 < 5$$

$$y - 3 + 3 < 5 + 3$$

$$y < 8, \quad y \in (-\infty, 8)$$

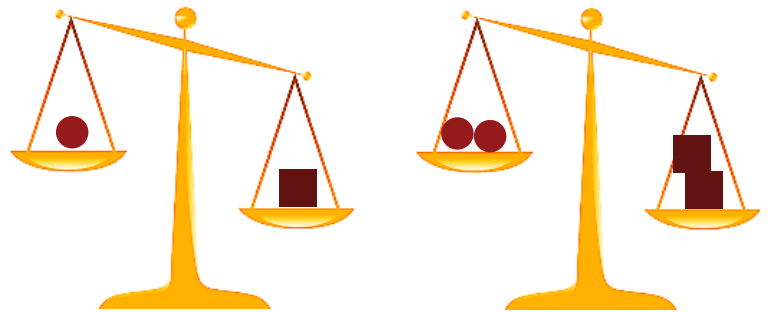
$$1. \quad x + 3 > -5$$



Now let's try to multiply or divide both part of the inequality by the positive number.

If  $x > 3$ , then  $2x$  will be greater than 6.

$$x > 3, \quad 2x > 6$$



If  $x > 3$  what can we tell about  $-x$  ?

$$-x \quad 3 \cdot (-1)$$

2.  $x + 3 > 5x - 5$

3.  $4x - 3 \neq 0$

4.  $3(x - 1) < 5x + 9$

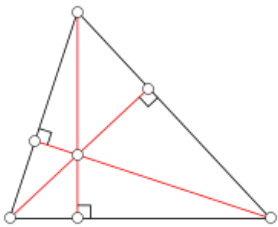
5.  $2x - 1 > -x + 3$

6.  $|x| > 8$

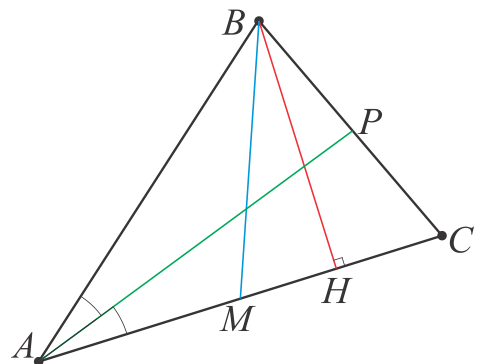
## Geometry.

### Special segments of a triangle.

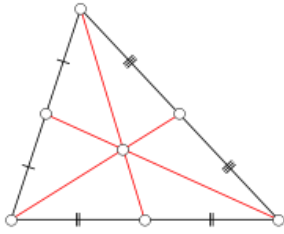
From each vertices of a triangle to the opposite side 3 special segment can be constructed.



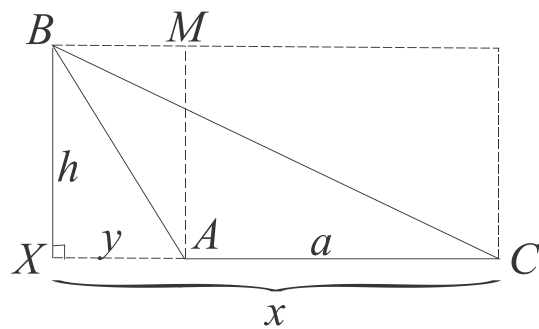
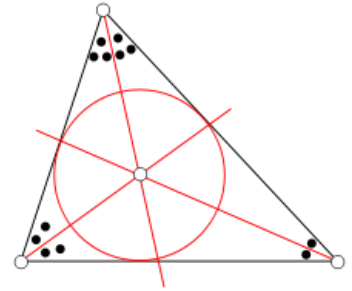
An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.



An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.



A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.



For an obtuse triangle, for one out of the three heights, it is not so obvious.

$$S_{\Delta XBC} = \frac{1}{2} h \times x, \quad S_{\Delta XBA} = \frac{1}{2} h \times y$$

$$\begin{aligned} S_{\Delta ABC} &= S_{\Delta XBC} - S_{\Delta XBA} = \frac{1}{2} h \times x - \frac{1}{2} h \times y \\ &= \frac{1}{2} h \times (x - y) = \frac{1}{2} h \times a \end{aligned}$$