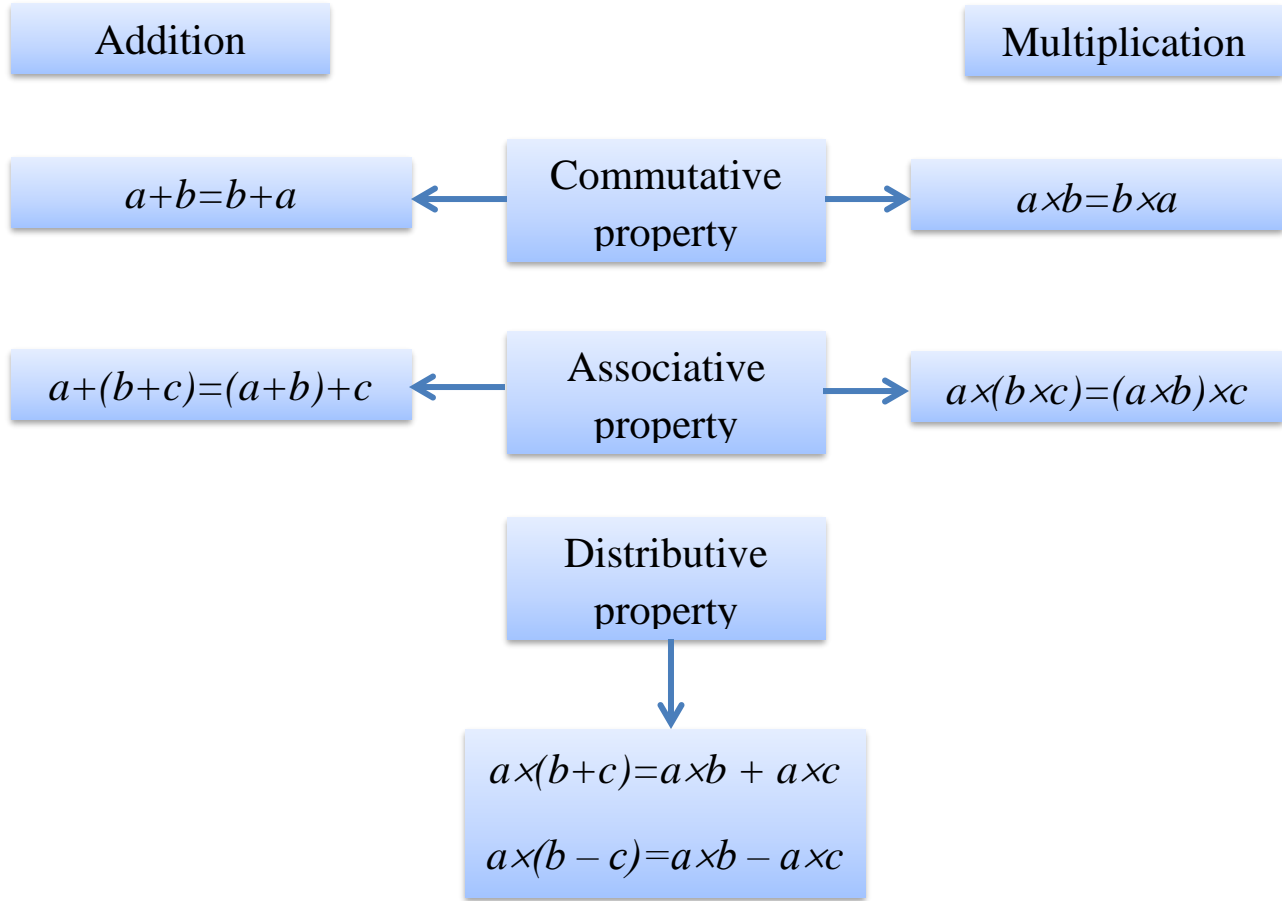
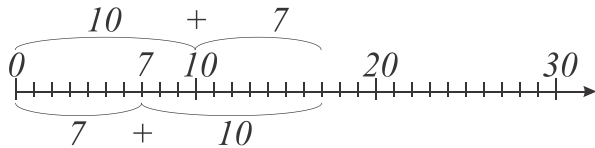


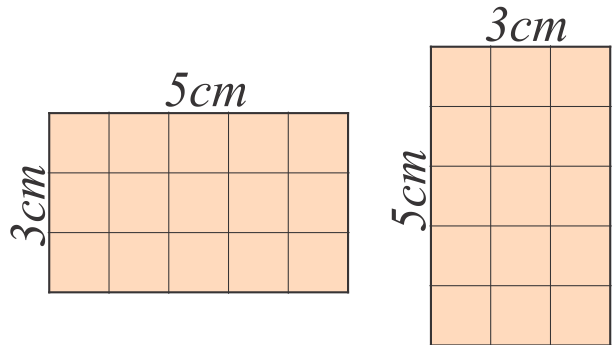
# Algebra



Commutative and associative properties are intuitively easy to understand.

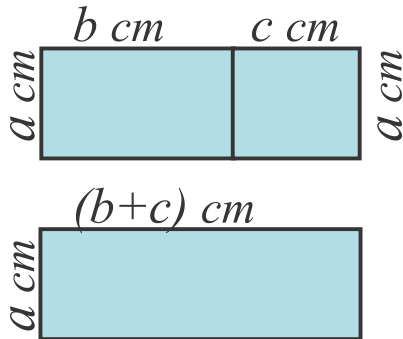


$$S = 3\text{cm} \times 5\text{cm} = 5\text{cm} \times 3\text{cm} = 15\text{cm}^2$$



Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes? Is there any difference between these 2 expressions:

$$5 \times (10 + 8) \text{ or } 5 \times 10 + 5 \times 8$$



Let's consider this in another example: The combined area of these 2 rectangles is

$S = a \times b + a \times c$  but the rectangle with one side  $a$  cm and the other  $(b+c)$  cm will have exactly the same area.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.

If  $a$  and  $n$  are natural numbers, the result of division operation of  $a \div n$  will be a quotient  $c$ , such that

$$a = b \times c + r$$

Where  $r$  is a remainder of division  $a \div b$ . If  $r$  is 0, then we can tell that  $a$  is divisible by  $b$ . If we want to divide  $m$  by 15, what numbers we can get as a remainder?

$$a : b = c$$

*dividend*     *divisor*     *quotient*

$$a = b \cdot c + r$$

*dividend*     *divisor*     *quotient*     *remainder*

### Exercises.

1.
  - a. The remainder of  $1932 \div 17$  is 11, the remainder of  $261 \div 17$  is 6. Is  $2193 = 1932 + 261$  divisible by 17? Can you tell without calculating and dividing?
  - b. Find all natural numbers such that upon division by 7 they give equal quotient and remainder.
2. Factorize (represent as a product of 2 or more multipliers) the following expressions:

*Example:*  $3 \times 5 + 3 \times 7 = 3 \times (5 + 7)$

- a.  $2 \times 3 + 2 \times 5 =$
- b.  $3x + 3y =$
- c.  $5a + 5b + 5c =$
- d.  $ab + ac =$
- e.  $ma - mb =$
- f.  $ds + dk - dl =$

3. Andrew prepares for an ironman competition. For that he swims for 37 minutes every day during 256 days and also he runs for 63 minutes every day during 256 days. How many minutes does he spend doing sports?

4. Rewrite the following expression without parenthesis:

- a.  $2 \times (a + b) =$
- b.  $a(x + y) =$
- c.  $(a + 2) \times 5 =$

5. Even or odd number will be the sum and the product of

- a. 2 odd numbers
- b. 2 even numbers
- c. 1 even and 1 odd number
- d. 1 odd and 1 even number

Can you explain why?

6. Solve the following equations:

$$2x - 4 = x + 8$$

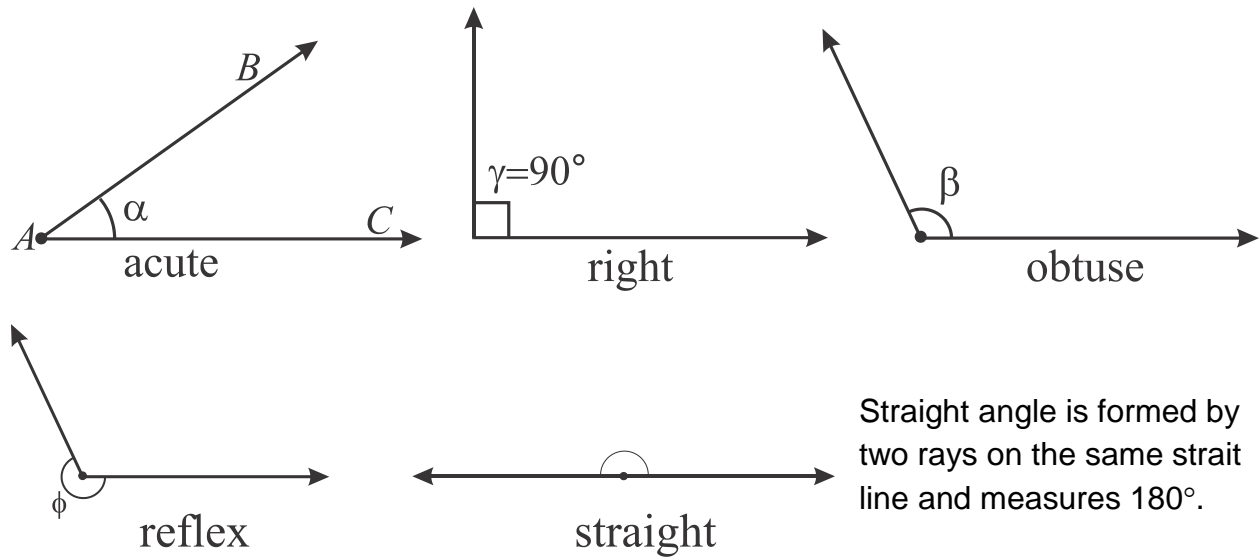
$$\frac{1}{2} + y = 3$$

$$z - \frac{1}{3} = \frac{2}{3}$$

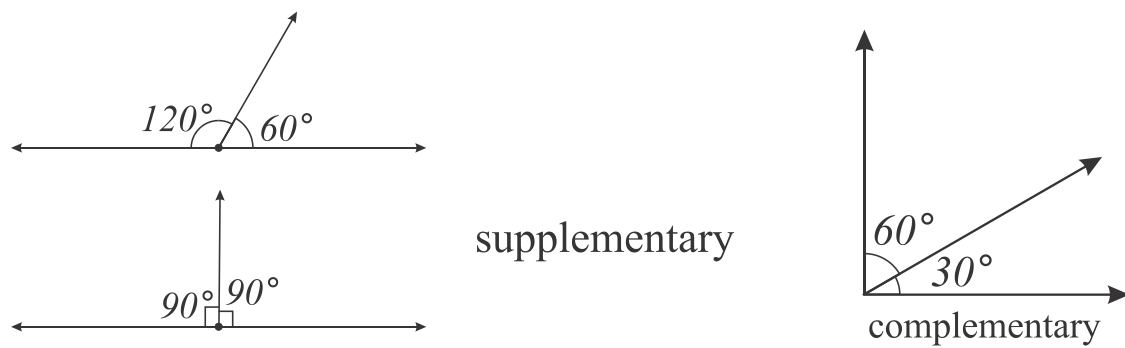
## Geometry.

An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter:  $\angle ABC$ ,  $\alpha$ . Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:

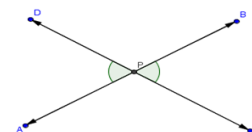


Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form a straight angle they are called supplementary; if they form a right angle then they are called complementary.

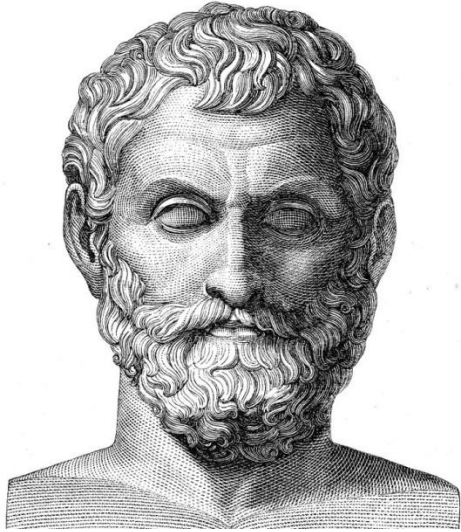


An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

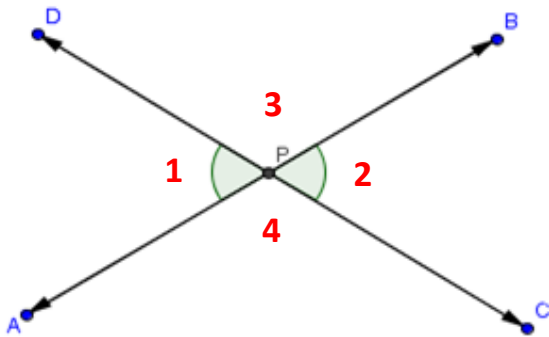


A historical legend tells us that when **Thales of Miletus** (624-546 BC), a Greek philosopher and mathematician, visited Egypt he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time. (Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).

Let's draw vertical angles: 1 and 2; 3 and 4 are two pairs of vertical angles.

**Vertical angles theorem: Vertical angles are equal.**

**Proof:**



$\angle 1 + \angle 3 = 180^\circ$  because they are supplementary by construction.

$\angle 2 + \angle 4 = 180^\circ$  because they are supplementary also by construction.

$$\angle 1 + \angle 3 = \angle 2 + \angle 3 \quad \text{Using substitution property of equation}$$

$$\angle 1 = \angle 2 \quad \text{Using subtraction property of equation}$$

We proved that if 2 angles are vertical angles then they are equal. Can we tell that if 2 angles are equal then they are vertical angles?

**Problem:** 4 angles are formed at the intersection of 2 lines. One of them is  $30^\circ$ . What is the measure of 3 others?