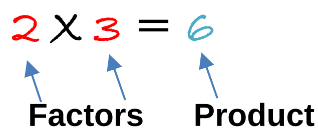
Math 4. Classwork 5.

Factors

"Factors" are the numbers you multiply together to get another number:



Factorization.

Factorization is breaking(decomposing) a number by its factors. It is the same as representing a number as a product of two or more other numbers, for example:

$$40 = 4 \times 10, \quad 36 = 6 \times 6$$

- Factorize:

$15a + 15b =$	$10x + 2y =$
$36w - 6 =$	$100 - 25x =$

- Compute using most convenient way: $12 \cdot 17 + 35 \cdot 13 + 17 \cdot 23 =$

- Can any natural number be represented as a product of two or more numbers besides 1 and itself?

Prime numbers

Prime numbers are building blocks of all composite numbers. Prime number can only be divided by 1 or itself, so it cannot be factored any further. Positive integers other than 1 which are not prime are called composite numbers.

- Can you think of another even number that is prime?

- Is 16 a prime?
- Is 51 a prime?
- How many prime numbers are out there?

Eratosthenes (c.276–194 bc) was a Greek scientist who was the first to measure the Earth's circumference using geometry. Eratosthenes produced a reliable, logical method for finding prime numbers: **The Sieve of Eratosthenes**. It does so by crossing out composite numbers (not primes). Composite numbers are multiples of each prime, starting with the multiples of 2.

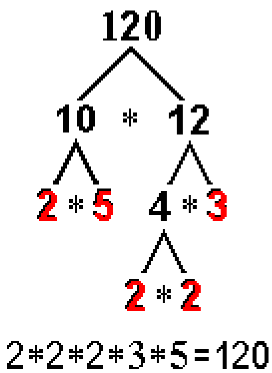
The Sieve of Eratosthenes

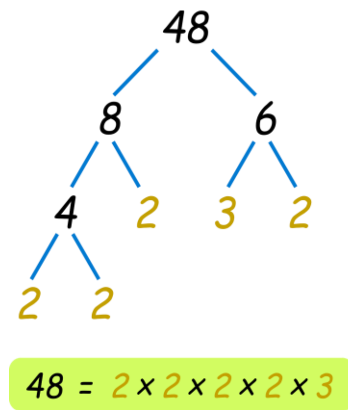
~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Prime <u>factorization</u> is finding **which prime numbers** multiply together to make the original number

There is only one (unique!) set of prime factors for any natural number.

Prime factorization process





- Find all prime factors of the following numbers:

  66 28 128 555 1233

Greatest Common Factor (Divisor)

The largest number that divides two given integers without remainder

To find GCF of two integers 24 and 40:

1. Prime factorize 24=6x4=2x3x2x2 40 =4x10=2x2x5x2
2. Find pairs 2x2x2=8
3. Both numbers are divisible by 8. GCF(24, 40)=8

- Find GCF (GCD) of

  42 And 45 81 94 and 125

Word Problem 1

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummy worms. What is the largest number of kids between whom the Jonson family can divide both kinds of candy evenly?

To solve this problem we have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both 168 and 180.

Let's look at a set of all prime factors of 168 and 180.

For 168 = 2x2x2x3x7. Any of these numbers as well as any of their products can be a divisor of 168.

For 180= 2x2x3x3x5.

These two sets have common elements. It means that both numbers are divisible by any of these common elements and any their products.

The greatest common factor (GCF) is the product of all common elements 2x2x3=12. So the candy can be divided evenly between 12 kids.

$$168 \div 12 = 14$$
$$180 \div 12 = 15$$

Least Common Multiple (LCM)

LCM is the smallest number that has given integers as factors. LCM is the smallest number that is a multiple of each of given integers.
To find LCM of two integers 24 and 40 you can find their multiples:

Multiples of 24	Multiples of 40
24x1=24	40x1=40
24x2=48	40x2=80
24x3=72	40x3=120
24x4=96	40x4=160
24x5=120	40x5=200

Or you can Prime factorize:

1. Prime factorize 24=6x4=2x3x2x2   40 =4x10=2x2x5x2
2. Find factors that represent each integer 2x2x2x3x5=120
3. LCM (24, 40) =120

Find LCM of

8 and 12 | 15 and 18 and 21

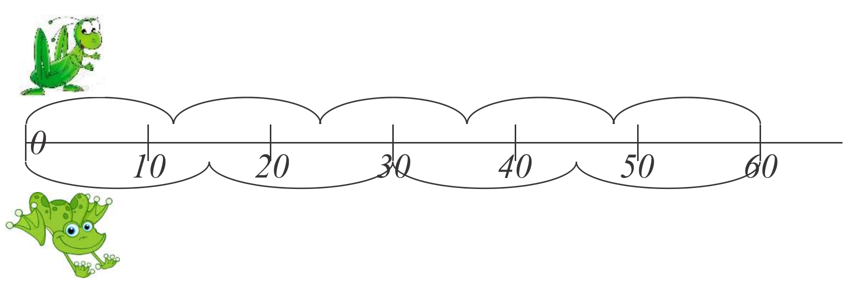
Word Problem 2

A grasshopper jumps the 12-centimeter distance each jump. A little frog jumps the 15 centimeters distance each jump. They start hopping from the point 0 and jump along the big ruler. What is the closest point on the ruler at which they both can land?

We should find a number that is divisible by both 12 and 15.

One method is to find multiples of 12 and multiples if 15

Multiples of 12	Multiples of 15
12x1=12	15x1=15
12x2=24	15x2=30
12x3=36	15x3=45
12x4=48	15x4=60
12x5=60	15x5=75
12x6=72	15x6=90



Another way to find out the answer is to use Prime factorization of 12 and 15:

12= 2x2x3

15=3x5

$2 \times 2 \times 3 \times 5 = 60$

We found that 60 is the smallest number, which is divisible by 12 and 15, LCM. So both frogs will be eventually landing on the 60 mark on our ruler.

Word Problem 3

In my class I will be giving quizzes several times a year. Each time I will include 12 questions on a quiz. Another teacher will be including 15 questions on each quiz in her class, but by the end of the year my and her students will all have the same number of questions. What is the least possible number of questions you and the other students will have to answer by the end of the year?

Word Problem 4

On a number line we marked numbers A, B, C, D. Can numbers A, B, C, D be prime numbers if number P is a prime number? Explain you answer. Can three consecutive numbers be prime numbers?