Math 4. Classwork 6.

Sets.

- I put a skirt, a book, a toothbrush, a coffee mug, and an apple into a bag. Can we call this collection of items a set? Do all these objects have something in common?

A set is a collection of objects that have something in common.

We can describe the members of a set by listing each member of the set:

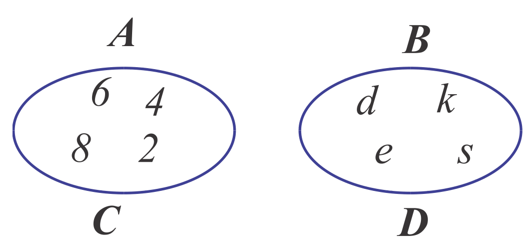
A = {2, 4, 6, 8}

B = {d, e, s, k}.

Or we can describe the members of a set by using a rule:

C is the set of four first even natural numbers.

D is the set of letters of the word "desk".



Venn diagram.

Two sets are equal if they contain the same elements. If we look closer on our sets ***A*** and ***C*** we can see that all elements of set ***A*** are the same as elements of set ***C*** (same goes for sets ***B*** and ***D*****).**

A*=*C and ***B*=*D***

If set ***A*** contains element '*2*', then we can tell that element '*2*' belongs to set ***A***. We have a special symbol to write it down in a shorter way: $2 \in A$

The set A does not contain 105--105$\notin$***A*.**

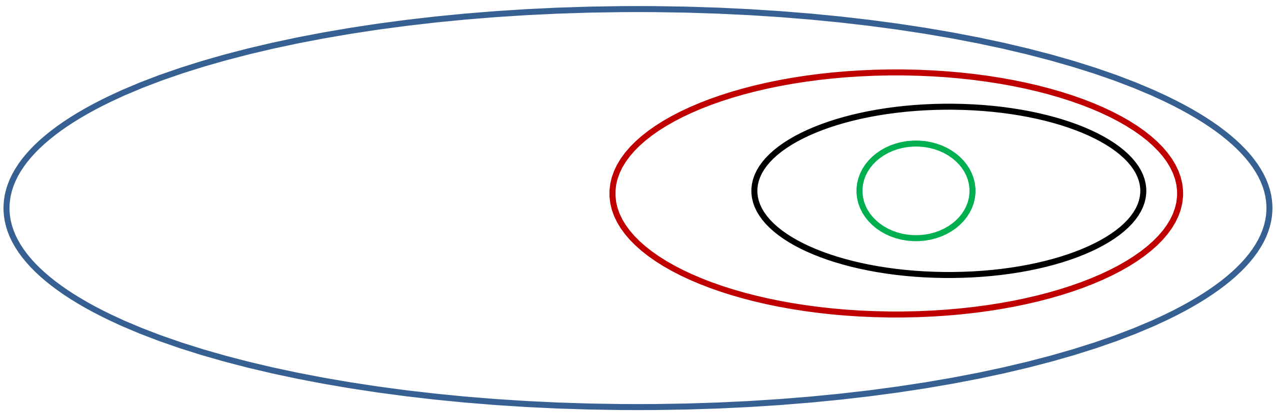
Let's define several sets.

Set ***W*** will be the set of all words of the English language.

Set ***N*** will be the set of all nouns existing in the English language.

Set ***Z*** will be the set of all English nouns which have only 5 letters.

Set ***T***={"table"}. On a Venn diagram below name all these sets:



A special symbol $\subset$ means "one set is a subset of another set". So our Venn diagram can be written:

$$T \subset Z \subset Y \subset W$$

If set ***V*** is defined as a set of all English verbs, can you draw a diagram for set ***V*** on the picture above? Can you tell subset of which set ***V*** is?

$V \subset$ _____

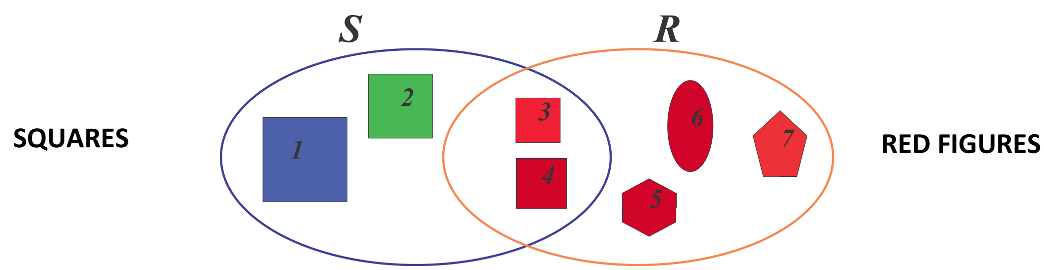
$V \not\subset$_____

$V \not\subset$_____

$V \not\subset$_____

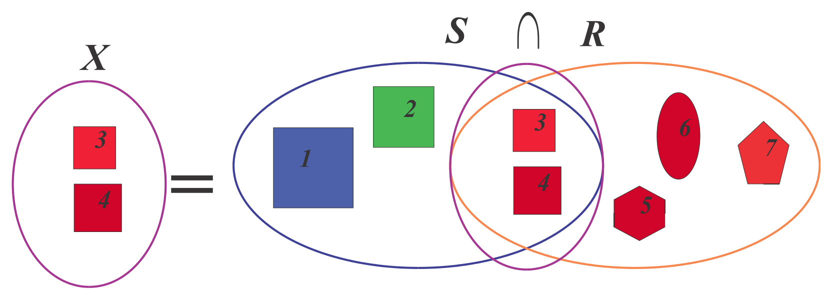
Set which does not have any element called an empty set in math people use symbol $\varnothing$).

When we define sets, a number of objects can belong to several sets at the same time. For example, on a picture below set ***S*** is a set of squares and a set ***R*** is a set of red figures.



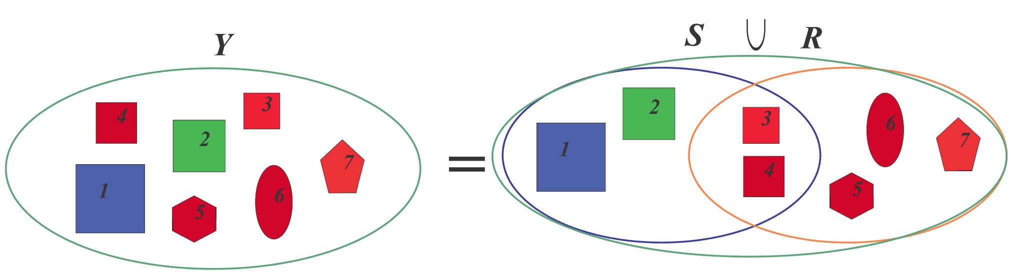
Figures 3 and 4 are squares and they are red, therefore they belong to both sets. The new set ***X*** contains elements that belong to the set ***S*** as well as to the set ***R***. Such set ***X*** is called an **intersection** of sets ***S*** and ***R*** and can be written using a symbol ∩.

$$X = S \cap R$$



If we combine all elements of ***S*** and ***R***, the new set ***Y*** would be a **union** of set ***S*** and ***R***. Using symbol ∪ we can easily write the sentence: Set ***Y*** contains all elements of set ***S*** and set ***R***:

$$Y = S \cup R$$



Which Way Does That "U" Go? Think of them as "cups": ∪ holds more water than ∩, right?

So Union ∪ is the one with more elements than Intersection ∩

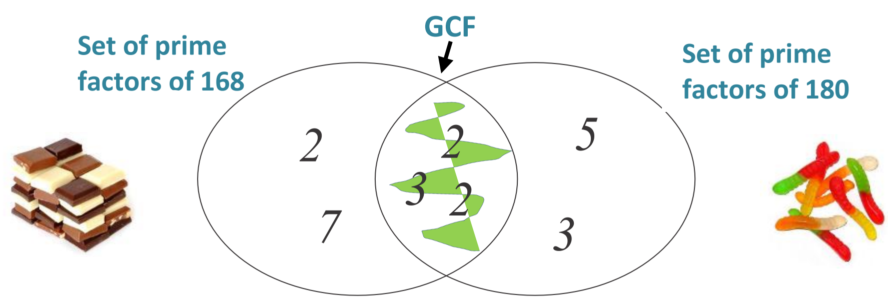
Exercises:

1. **Let's show GCF and LCM using Venn Diagram for our last week's problems:**

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly? We need to find GCF:

$$\textit{Prime factors of } 168 \textit{ and } 180 \textit{ are } 2\times2\times2\times3\times7 = 168;\ 2\times2\times3\times3\times5 = 180$$

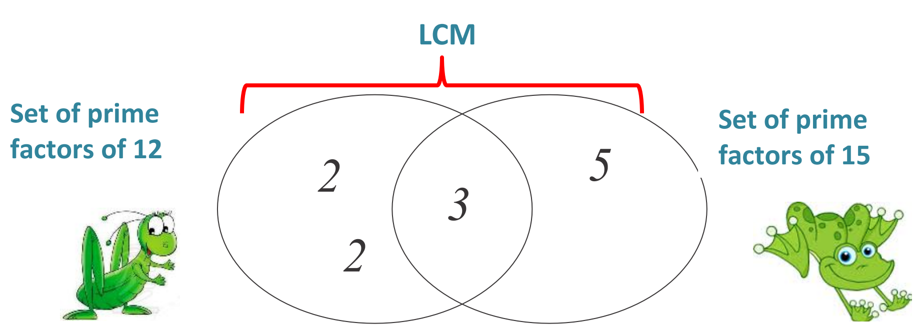
GCF is the intersection of the two sets of prime factors (the common set of prime factors)



A grasshopper jumps a distance of 12 centimeters each jump. A little frog jumps a distance of 15 centimeters each jump. They start hopping at the same time from the same point 0 and jump along the big ruler. What is the closest point on the ruler at which they can meet? What is the smallest number that is divisible by 12 AND 15? We need to find LCM:

$$\textit{Prime factors of } 12 \textit{ and } 15 \textit{ are } \quad 2\times2\times3 = 12;\ \ 3\times5 = 15$$

The number which we are looking for must be a product of prime factors of 12 AND 15, so it should be a union of two sets – set of prime factors of 12 and 15.



$2\times2\times3\times5 = 60$

60 is the smallest number, which is divisible by 12 and 15, LCM.

2. $\boldsymbol{A} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \boldsymbol{B} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$. Write the intersection $(\boldsymbol{A} \cap \boldsymbol{B})$ and the union $(\boldsymbol{A} \cup \boldsymbol{B})$ of these two sets.

3. There are 20 students in a Math class. 10 students like apples and 15 students like pears. Show that there are some students who like both apples and pears.

   - Assume that each student likes at least one of the fruits. (This means that each student like either apples, or pears, or both). How many students like both pears and apples?

   - Is it possible to determine if there are any students who do not like apples and do not like pears?

   - Which part of the diagram shows:" Those who like apples, but not pears"?

4. The same Math class (with 20 students) forms a soccer team and a basketball team. Every student signs up for at least one team:

   - 12 students play only soccer;
   - 2 students play both soccer and basketball;

   How many students play basketball only?

5. Students who participated in math coopetition had to solve 2 problems, one in algebra and another in geometry. Among 100 students 65 solved an algebra problem, 45 solved a geometry problem, 20 students solved both problems. How many students didn't solve any problem at all?

6. 240 students from New-York and Seattle attended a math camp. Of the total number of students, 125 were boys. 65 boys were from New-York. There were 53 girls from Seattle. How many students came from New-York?

Symbols to Remember

∈	element belongs to a set
∉	element does not belong to a set
⊂	one set is a subset of another set
⊄	one set is not a subset of another set
∩	intersection of two sets (elements that are in both sets)
∪	union of two sets **(**elements that are in either set)
∅	empty set