Math 5b: Homework 9

Power Rules

General notation (*n* is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \text{ (}n\text{ times)}$$

Special cases:

$a^0 = 1$	read: *a*-to-the-zero
$a^1 = a$	is just itself '*a*'
$a^2 = a \times a$	read: *a*-squared
$a^3 = a \times a \times a$	read: *a*-cubed

Properties:

$$(ab)^n = ab \times ab \times ab \times \ldots \times ab \text{ (}n\text{ times)}$$
$$(ab)^n = (a \times a \times a \times \ldots \times a) \times (b \times b \times b \times \ldots \times b) \text{ (}n\text{ times)}$$
$$(ab)^n = a^n \times b^n$$

Similarly:

$$a^n a^m = (a \times a \times a \ldots) \times (a \times a \times a \ldots) \text{ (}n\text{ and }m\text{ times, respectively)}$$
$$a^n a^m = a \times a \times a \ldots \times a \times a \text{ (}n+m\text{ times)}$$
$$a^n a^m = a^{n+m}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^n = \frac{1}{a^{-n}}$$
$$a^{-n} = \frac{1}{a^n}$$

Classwork

1. Solve the following equations:
   (a) $5 - x = -4 - 2x$

   (b) $7 - 2(1 - x) = -5$

   (c) $\frac{x+2}{x-1} = 3$

2. If you take half my age and add 7, you get my age 13 years ago. How old am I?

3. Simplify:

   (a) $\frac{(x^2y^2)x^3}{x^2y^5}$

   (b) $(3y^3 \cdot y^5)^2$

4. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$(Hint: use $(a \cdot b)^n = a^n b^n$ and $(a^n)^m = a^{mn}$)

5. How many cubic centimeters are there in one cubic kilometer? (1km = 1000m, 1m=100cm)

6. It is known that $2^{10} = 1024$, which is very close to 10^3. Use this to estimate the value of 2^{20}, 2^{32}

7. Evaluate:
   (a) $(x - 5)(2x + 1) =$
   (b) $(x + 7)(x^2 - 2x) =$

8. Solve:
   (a) $2^{-2} \cdot (2^2 + 4^2) =$
   (b) $6^3 \cdot (2^{-3} + 3^{-3} =$

9. One can measure temperature using either the Fahrenheit scale (common in the US and Britain) or the Celsius scale (in most other countries). The relation between the two is given by
   $\boldsymbol{C = \frac{5}{9}(F - 32)}$ [C in the temperature in Celsius, F – in Fahrenheit]
   (a) Is there a temperature which gives the same value on both scales (F = C)?
   (b) Is there a temperature which in Fahrenheit scale is twice as large as in Celsius (F = 2C)?

10. There are three buckets: 10 liters, 4 liters, and 3 liters. The 10-liter bucket is full of water. There is no other water available. Divide the water so that there is exactly 5 liters in the 10-liter bucket, 1 liter in the 3-liter bucket, and 4 liters in the 4-liter bucket. You may only pour back and forth between the three given buckets. Describe how to do that using a table below. First and last columns are done for you.

10-l bucket						5
4-l bucket						4
3-l bucket						1