Math 5b: Homework 20
Homework #20 is due March, 25

Congruence

In general, two figures are called congruent if they have the same shape and size. We use the symbol ≅ to denote congruent figures: to say that M_1 is congruent to M_2, one writes $M_1 \cong M_2$.

The precise definition of what the "same shape and size" means depends on the figure. Most importantly, for triangles it means that corresponding sides are equal and corresponding angles are equal: $\Delta ABC \cong \Delta A'B'C'$ is the same as:

$$AB = A'B', \quad BC = B'C', \quad AC = A'C',$$

$$\angle A = \angle A', \quad \angle B = \angle B', \quad \angle C = \angle C'.$$

Note that for triangles, the notation $\Delta ABC \cong \Delta A'B'C'$ not only tells that these two triangles are congruent, but also shows which vertex of the first triangle corresponds to which vertex of the second one. For example, $\Delta ABC \cong \Delta PQR$ is not the same as $\Delta ABC \cong \Delta QPR$.

Congruence tests for triangles

By definition, to check that two triangles are congruent, we need to check that corresponding angles are equal and corresponding sides are equal; thus, we need to check 6 equalities. However, it turns out that in fact, we can do with fewer checks.

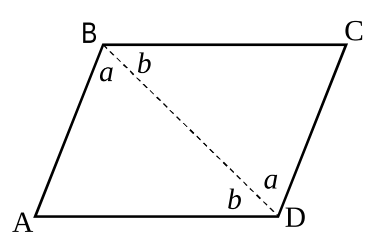
Rule 1 (Side-Side-Side rule). If $AB \cong A'B'$, $BC \cong B'C'$ and $AC \cong A'C'$ then $\Delta ABC \cong \Delta A'B'C'$.

This rule is commonly referred to as the ***SSS*** rule.

One can also try other ways to define a triangle by three pieces of information, such as two sides and an angle between them. We will discuss it next time. This rule – and congruent triangles in general – are very useful for proving various properties of geometric figures. As an illustration, we prove the following useful result.

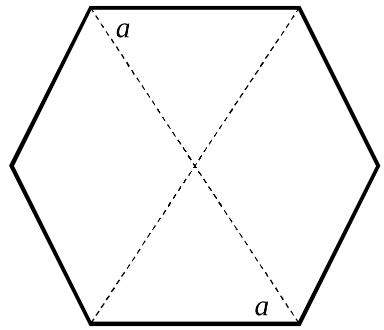
Theorem. Let $ABCD$ be a quadrilateral in which opposite sides are equal: $AB = CD$, $AD = BC$. Then $ABCD$ is a parallelogram.

Proof. Let us draw a diagonal BD. Then triangles ΔABD and ΔCDB are congruent by ***SSS***; thus, the two angles labeled by letter a in the figure are equal; also, the two angles labeled by letter b are also equal. Thus, lines BC and AD are parallel (alternate interior angles!). In the same way we can show that lines AB and CD are parallel. Thus, $ABCD$ is a parallelogram. (It is also true in the opposite direction: in a parallelogram, opposite sides are equal. We will prove it next time.)



Homework

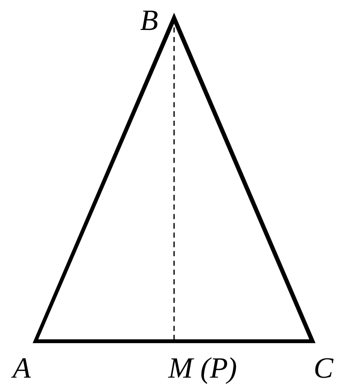
1. An *n-gon* is called *regular* if all sides are equal and all angles are also equal.
   (a) How large is each angle in a regular hexagon (6-*gon*)?
   (b) Show that in a regular hexagon, opposite sides are parallel. (This is the reason why this shape is used for nuts and bolts).
   [Hint: show that each of the angles labeled by the letter *a* in the figure is equal to 60^o, and then use the theorem about alternate interior angles.]



2. Let ABC be a triangle in which two sides are equal: $AB = BC$ (isosceles triangle). We proved in class that if M is the midpoint of the side AB, i.e. $AM = MB$, then
   - ü triangles AMC and BMC are congruent.
   - ü angles A and B are equal
   - ü angle $AMC = 90$ degrees

   So, in an isosceles the median is also a height.

   a) Please, review your notes and prove the above 3 points again!



   b) Can you prove the following: If in the triangle ΔABC $\angle A = \angle B$ and the point P of the side AB is such that CP is a height (angle $APC = 90^o$) then this triangle is isosceles?

3. Simplify the expressions:
   (a) $x - (1 + 5x) =$
   (b) $2x - (3x^2 + x - 1) + (2 + 2x - x^2) =$
   (c) $3x(-2xy) =$
   (d) $(y - 5)(y - 1) - (y + 2)(y - 3) =$
   (e) $3(x - 1)^2 - 3x(x - 5) =$