## SchoolNova, Math 5c <br> Homework 4 <br> More Numbers and Fractions <br> October 15, 2017

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a $*$. If unable to solve a problem, please present your thoughts and partial solution. The problems, which require some algebra are also marked with a $*$, this will be covered during the course.

1. In the following sequence, every term, starting with the fourth term is the sum of the three terms before it.

$$
1,1,2,4,7,13,24, \ldots
$$

Is the $100^{\text {th }}$ term in the sequence even or odd?
2. Five lamps $L_{1}, L_{2}, L_{3}, L_{4}$ an $L_{5}$ are arranged in a row as shown below. Each lamp has its own switch. All five lamps are initially off. Ben starts to turn each switch from $L_{1}$ through $L_{5}$, in order, and he repeats the pattern, always from $L_{1}$ through $L_{5}$, until he turns the switches 126 times. Which lamps are on in the end?

$$
\begin{array}{lllll}
L_{1} & L_{2} & L_{3} & L_{4} & L_{5}
\end{array}
$$

3. Is the expression $1 \times 2+3 \times 4+5 \times 6 \ldots 99 \times 100$ even or odd?
4. $1^{2}+2^{2}+3^{2}+\ldots+25^{2}=5525$ and
$2^{2}+4^{2}+6^{2}+\ldots+50^{2}=N$.
Find the value of $N$.
5.     * Show that if $n$ and $m$ are divisible by $d$, then each of the following numbers is divisible by $d$ :
(a) $n+m$ (b) $5 n+3 m$
6. $*$ If $x$ and $y$ are positive consecutive even integers, with $y>x$, which of the following is equal to $y^{2}-x^{2}$ ?
(a) 2 x
(b) $4 x$
(c) $2 x+2$
(d) $2 x+4$
(e) $4 x+4$
7. A boy had a bag of apples. He gave $1 / 2$ of them to his parents, $1 / 5$ to his brother, $1 / 4$ to his sister, and the last apple he ate himself. How many apples did he originally have?
8. If $0<a<b$, which of the following is greater than $b / a$ ?
(a) 1
(b) a/b
(c) $1 /(\mathrm{b} / \mathrm{a})$
(d) $\mathrm{b} /(2 \mathrm{a})$
(e) $2 \mathrm{~b} / \mathrm{a}$
9. If $x<x^{2}<x^{3}$, which of the following could be the value of $x$ ?
(a) -2
(b) $-\frac{1}{2}$
(c) $\frac{2}{3}$
(d) 1
(e) $\frac{3}{2}$
10. $*$ Find the sum:

$$
\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\frac{1}{3 \times 4} \cdots \frac{1}{11 \times 12}
$$

(There is a clever trick)

