

Topics discussed

January 21, 2018

1. Theorem: any rational number is a finite or repeating decimal. The way we proved is using **Pigeonhole principle**.



Pigeonhole principle states that if n items are put into m pigeonholes with $n > m$, then at least one pigeonhole must contain more than one item.

2. Operations with powers:

$$a^n = a \cdot a \cdots a \text{ (} n \text{ times)}$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

3. Power Properties and Scientific Notation

$$5.12 \times 10^6 = 5.12 \times 1000000 = 5120000$$

$$1.2 \times 10^4 \times 3 \times 10^8 = 1.2 \times 3 \times 10^4 \times 10^8 = 3.6 \times 10^{12}$$

4. We reviewed that $1\text{L}=1\text{dm}^3$, $1\text{mL}=1\text{cm}^3$.

HOMEWORK 13,
January 21, 2018

1. Solve the following equations:

$$\text{a) } \frac{5y-12}{3-2y} = 2 \qquad \text{b) } \frac{8-2x}{3x-1} = 3 \qquad \text{c) } \frac{3x+2a}{2a-5x} = -1$$

2. Let $a = 2 \cdot 10^8$, $b = 10^5$, compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$.

$$1L = _? _ cm^3 = _? _ mm^3 _? _ = m^3 _? _ = km^3$$

3. It is known that $2^{10} = 1024$, which is very close to 10^3 . Using this, can you **estimate** what is the value of 2^{20} ? 2^{32} ?

4. Consider the sequence $7, 7^2, 7^3, \dots, 7^n, \dots$

(a) Show that there will be two numbers in this sequence which have the same last two digits. [Hint: pigeonhole principle!]

(b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.

5. (from 101 puzzles in thought and logic, by C. R. Wylie) Clark, Jones, Morgan, and Smith are four men whose occupation are butcher, druggist, grocer, and policeman, though not necessarily in that order.

Clark and Jones are neighbors and take turns driving each other to work.

Jones makes more money than Morgan.

Clark beats Smith regularly at bowling.

The butcher always walks to work.

The policeman doesn't live near the druggist.

The only time the grocer and the policeman ever meet is when the policeman arrested the grocer for speeding.

The policeman makes more money than the druggist or the grocer.

What is each man's occupation?

6. **(Optional breakfast suggestion)



It is easy to cut a bagel in two pieces so that each piece looks like a ring (not perfect, of course). But can you cut a bagel so that you get two linked rings?

<https://www.youtube.com/watch?v=dN8AwGUaqDA>

<https://www.youtube.com/watch?v=jBVzQu7Vw5E>

Be careful with a knife !