MATH 6: THE MATH BATTLE

1. On a piece of paper, Laura has drawn a green square and Jessica — a red square. Is it always possible to cut the paper (by making a single cut along a straight line) so that each square is cut into two pieces of equal area?

2. A king puts his 5 daughters in 5 rooms. To confuse a knight who is trying to find the youngest daughter, the king's advisor suggested that the daughters change rooms every day: in each room there should be an instruction saying "If you spend the night in this room, next night you should spend in room number...".[The instruction does not change from day to day.]

   "But then", said the king, "after 5 nights at most, all of them will be back to original rooms!". "Not necessarily", says the advisor.

   Can you devise a scheme so that the original room assignment is not repeated until day 7 (i.e., on days 1, 2, 3, 4, 5, 6 the room assignments are different)?

3. In each square of 9×9 board there is a pawn. Can you move each pawn to one of the adjacent squares so that after the move, there is again one pawn in each square of the board?

4. A traveler meets two brothers, Andrew and Bob. It is known that one of them always lies. "Are you Andrew?" he asks of the first brother. "Yes", answers the brother. Then the traveler asks the second brother the same question and gets an answer – but we do not know what the answer is. However, after hearing the answer the traveler immediately knew which brother was Andrew and which was Bob. Can you tell which brother is Andrew?

5. Suppose I have a 1×10 strip of paper. I want to tile it with 1×1 ("squares") and 1×2 ("dominoes") pieces of paper. In how many ways can I do this? [Hint: start off with smaller strips — and try to notice a pattern!] Can you generalize it to a $1 \times n$ strip?

6. Can you divide a given triangle into 4 smaller triangles, so that any two of them are adjacent to each other? [Note: two triangles are called *adjacent* if they share an entire edge or a part of an edge.]