# Math 6b/c: Homework 5

Homework #5 is *due October 29*. Please, write clearly which problem you are solving and show *all steps* of your solution.

In problems 1 and 2, you need to (a) write the obvious conclusion from the given statements; and (b) justify the conclusion by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas (denoting which are used by letters A, B, … connected by logical operations *OR*, *AND*, …).

1. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore …

2. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore …

3. Using only *AND*, *NOT*, and *OR*, produce a three-input *AND* circuit, i.e. the output is F unless all three inputs are a T. (You do not have to use all three circuit elements.)

4. The diagram right shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output is on the right). Can you determine for which values of inputs the LED will light up? (Hint: this is the same writing a truth table for some formula…)
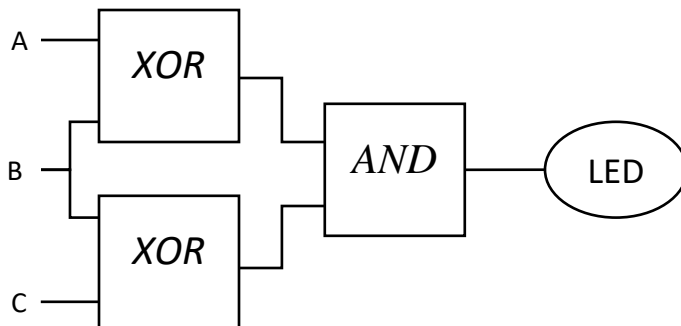
Note that the wires connecting each of the chips and LED to the power source are not shown.

5. Let us consider a new logical operation, called *NAND*, which is defined by the following truth table:

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that A *NAND* B is equivalent to A *NAND* B (this explains the name *NAND* for 'not and')
(b) Show that A *NAND* A is equivalent to *NOT* A
(c) Write the truth table for $(A$ *NAND* $B)$ *NAND* $(A$ *NAND* $B)$
(d) Write the truth table for $(A$ *NAND* $A)$ *NAND* $(B$ *NAND* $B)$
(e) Show that any logical formula which can be written using *AND*, *OR*, *NOT* can also be written using only *NAND*.

6. On the island of knights and knaves, you meet two inhabitants: X and Y. X says 'Y is a knave'. Y says 'X is a knave'. Who is a knight and who is a knight?