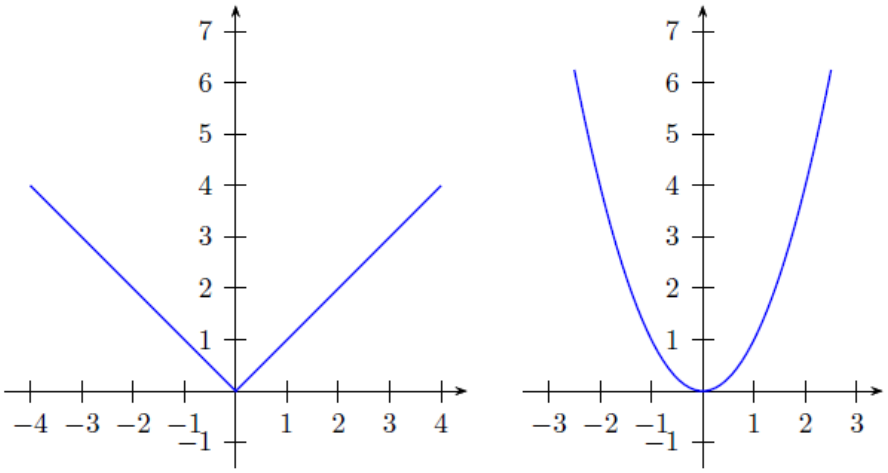
Math 6b/c: Homework 14
Homework #14 is due January 28.

Graphs

Generally, a graph of function y = f(x) is some line in the $x - y$ plane. If one has two graphs y = f(x) and y = g(x) one can find intersection points of corresponding graphs by solving the system of equations. For example, the intersection point of two straight lines y = x+2 and y = −x is the point (−1, 1) as x = −1 and y = 1 satisfy both of these equations; that is the point (−1, 1) lies simultaneously on both straight lines.

Graphs of y = |x| and y = x^2

The figures below show graphs of functions y = |x| and y = x^2; the latter graph is called a *parabola.*



And here is what we can do to draw a graph of any parabola of the *standard form* $y = ax^2 + bx + c$. You can verify the following identity yourself:

$ax^2 + bx + c = a(x - h)^2 + k$ (*vertex form*), where $h = -\frac{b}{2a}$ h and $k = -\frac{b^2-4ac}{4a}$.

You can convert from standard to the vertex form by using the coefficients *a*, *b*, *c* to obtain *h* and *k*, and then re-write the graph equation into the vertex form.
For example: $y = x^2 + x$ can be converted into $y = (x + \frac{1}{2})^2 - \frac{1}{4}$

The result will be a parabola obtained by stretching the usual parabola vertically by factor a (if a < 0, this means flipping it upside down and then stretching by |a|) and then moving it so that the vertex will be at point (h, k). In particular, the branches go up if a > 0 and down if a < 0.

Obviously, the parabola either intersects y = 0 (x-axis) at two points, does not intersect it, or touches y = 0 at a single point. Correspondingly, the quadratic equation has two roots, no roots or one root respectively. One can easily check that this corresponds to D > 0, D < 0 and D = 0 respectively, where D = b^2 − 4ac.

Homework

1. Find the equation of the line which passes through point (3,4) and has a slope +2. (Hint: you only need to find the intercept and write y = ax+b)

2. Find the equation of the line through points (-2, 0) and (0,2).

3. Sketch the graph of the functions: $y = |x + 1|$ and $y = -x + 0.25$. How many solutions do you think the following equation has?

   $$|x + 1| = -x + 0.25$$

   Note: you are not asked to solve the equation – just answer how many solutions there are.

4. Find the intersection point of a line $y = ¼ x^2$ and a line $y = 2x+1$. Sketch or draw the graphs. (Hint: construct a system of equations and solve).

5. Sketch/draw graphs of the following functions:
   a) $x + y = 2$ b) $y = |x - 5| + 1$ c) $y = |x + 1| + |x - 2|$
   d) $y = |x + 1| + |x + 2| + |x + 3|$

6. Convert the function $y = -x^2 + 4x - 3$ from standard to vertex form.
   Sketch/draw the function. Does the graph intersect the y = 0 line (x-axis). Does the number of roots correspond to the D-value?