Math 6b/c: Homework 19
Homework #19 is due March 11.

Factorials and permutations

If we are to choose *k* objects from a collection of *n* so that a) order matters and b) no repetitions are allowed, then there are

$$n(n-1)\ldots(n-(k-1)) \qquad (k \text{ factors})$$

ways to do it.

If we take $k = n$, it means that we are selecting (one-by-one) all *n* objects, which gives the number of possible ways to order *n* objects:

$$n! = n(n-1)(n-2)\ldots(2)(1)$$

n! is read as '*n* factorial'.

For example: there are 52! ways to mix cards from a regular deck of cards.
Note that the number *n!* grows very fast: 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 620, and note that we define 0! = 1.

Homework

In all the problems that ask you to compute something, it is sufficient to write an expression for the answer, e.g., $1/2^{11}$, it is not necessary to perform the multiplication.

1. About 1/6 of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? that none have blue eyes? that at least one has blue eyes?

2. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

3. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

4. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so that the pattern matches (this kind of puzzles is quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination ***one second***, how long will it take you to try them all?

5. At a fair, you are offered to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about $5). Unfortunately, it is impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.

(a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?
(b) Same questions for five balls.