Classwork 2 + Homework 2
Vectors

Math 7a

September 24, 2017

We all know that any point (A) in a plane can be written down as a pair of numbers or its coordinates: along x axis (A_x) and along y axis (A_y).

We will now learn to use a pair of points in plane: a vector. A vector in 2-dimensional plane is simply a line that has a direction. It is a line segment ($\overrightarrow{AB}$) of a particular length with endpoints (A and B). Specifying coordinates of points A: (A_x, A_y) and B: (B_x, B_y) tells us exactly what the vector looks like.

Vector $\overrightarrow{AB}$'s coordinates are the difference between coordinates of its head B and tail A:

$$\overrightarrow{AB} = (B_x - A_x, B_y - A_y)$$

Length of a vector can be found using Pythagorean theorem. We can draw a right triangle with sides $B_x - A_x$ and $B_y - A_y$. Then length of the hypothenuse will give us the length of vector $\overrightarrow{AB}$. This can be written down as:

$$|\overrightarrow{AB}| = \sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}$$

FIGURE 1: VECTOR AND ITS ENDPOINTS

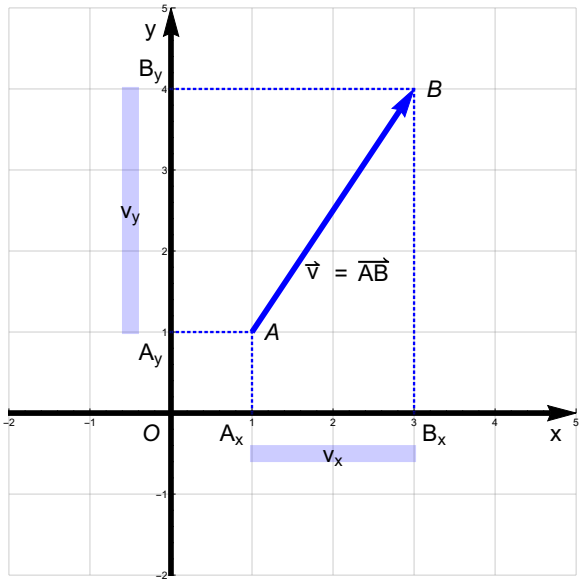
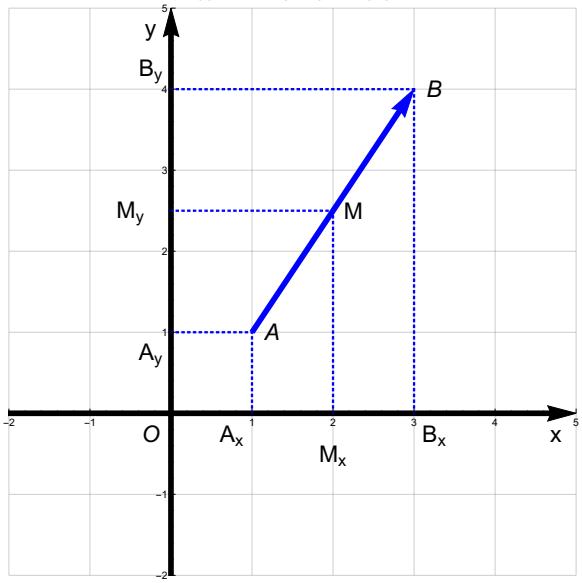


FIGURE 2: MIDPOINT OF A VECTOR



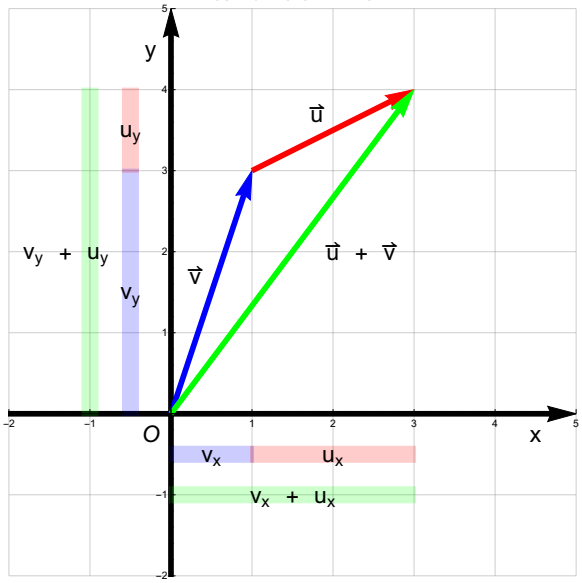
Midpoint of a vector $\overrightarrow{AB}$ is the point located exactly halfway between the tail and the head of the vector. Midpoint is usually denoted as M and can be found by averaging the coordinates of the vector's endpoints along each axis:

$$\overrightarrow{M} = (\frac{1}{2}(B_x + A_x), \frac{1}{2}(B_y + A_y))$$

Just like numbers, we can add vectors. The addition of vectors is done by adding the individual components of the vectors. If vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are given by $\overrightarrow{u} = (v_x, v_y)$ and $\overrightarrow{u} = (u_x, u_y)$, then their addition gives us a vector:

$$\overrightarrow{u + v} = (v_x + u_x, v_u + u_y)$$

FIGURE 3: VECTOR ADDITION



1. We are given coordinates of 2 points: $A = (A_x, A_y)$ and $B = (B_x, B_y)$. How can we write down the vector whose endpoints are A and B?

2. Find the midpoint of a vector given by its endpoints:
   (a) $(10, 10)$ and $(0, 0)$
   (b) (x, y) and $(12, 10)$

3. If endpoints of a vector are $C = (1, 1)$ and $D = (5, 6)$, find the following:
   (a) coordinate of the midpoint M between C and D
   (b) coordinates of points $C + \overrightarrow{CD}$, $C + 2\overrightarrow{CD}$, $C - \overrightarrow{CD}$, and $C + \frac{1}{2}\overrightarrow{CD}$.

4. If endpoints of a vector are $C = (C_x, C_y)$ and $D = (D_x, D_y)$, find coordinates of points $C + \overrightarrow{CD}$, $C + 2\overrightarrow{CD}$, $C - \overrightarrow{CD}$, and $C + \frac{1}{2}\overrightarrow{CD}$.

5. 3 points of a parallelogram $ABCD$ are $A = (1, 1)$, $B = (2, 5)$, and $C = (6, 3)$. What is the coordinate of:
   (a) 4th point D of the parallelogram *(you are free to pick one of several possible 4th points)*
   (b) Midpoint of side $\overrightarrow{CD}$
   (c) "Center" of the parallelogram?

6. We are given 2 points: $A = (2, 2)$ and $B = (10, 6)$. We also know that point C divides the line segment $\overrightarrow{AB}$ into segments $\overrightarrow{AC}$ and $\overrightarrow{CB}$, whose lengths have proportion $3 : 1$, i.e., $|\overrightarrow{AC}| = 3|\overrightarrow{CB}|$. Find the coordinate of point C.

7. We are given 2 points A and B as well as a point C, which splits the segment $\overrightarrow{AB}$ in half. Note that this time, we don't have the coordinates specified. Show that:

$$\overrightarrow{OC} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}$$

8. A triangle ABC with vertices A, B, and C is given.
   (a) Calculate $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.
   (b) Is it true that $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ is equal to $\overrightarrow{BA} + \overrightarrow{CB} + \overrightarrow{AC}$?

9. We are given a rectangle $ABCD$ as drawn in figure below. The midpoints of sides $\overrightarrow{BC}$ and $\overrightarrow{CD}$ are M and N, respectively.
   (a) Express the vectors $\overrightarrow{BC}$ and $\overrightarrow{DC}$ using only vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$. *(Hint: this is easy.)*
   (b) Express the vectors $\overrightarrow{AM}$ and $\overrightarrow{AN}$ using only vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.
   (c) Finally, express the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ using only vectors $\overrightarrow{AM}$ and $\overrightarrow{AN}$.

FIGURE 4: MIDPOINTS OF SIDES