Homework 11
Quadratic Polynomial And Basic Parabola

Math 7a

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The graph of $y = x^2$ is referred as the basic parabola, and its general coordinates are (n, n^2).

Vertical translation of a parabola
When we translate the parabola vertically upwards or downwards, the y-value of each point on the basic parabola is increased or decreased, respectively. Thus, for example, translating the parabola upwards/downwards by 4 units, shifts the general point (n, n^2) to $(n, n^2 + 4)$ or $(n, n^2 - 4)$, respectively. The equation of this new parabola is thus $y = x^2 \pm 4$. The vertex of this parabola is now $(0, 4)$ or $(0, -4)$, respectively. It still has the same axis of symmetry.

Horizontal translation
When we translate the basic upside-down parabola to the right by 4 units the x-value becomes $n + 4$, the equation is $y = -(x-4)^2$ such that for the shifted point $n+4$ the y-value stays the same $-((n+4)-4)^2 = -n^2$. The graph of $y = -(x - 4)^2$ is congruent to the basic parabola, but is translated 4 units to the left. The vertex of this parabola is now $(4, 0)$. Its new axis of symmetry is the line $x = 4$.

Stretching a parabola
The basic parabola $y = x^2$ can have its arms stretched producing a new parabola that is not congruent to the original one. Thus the parabola $y = 5x^2$ is obtained from the parabola $y = x^2$ by stretching by a factor of 5 from the x-axis, that is, the y-values are increased by factor of 5.
In general any quadratic polynomial can be written by completing the square:

$$y = f(x) = a(x - h)^2 + k$$

and its turning point is $(x, y) = (h, k)$ or more precisely since

$$(x + \frac{b}{2a})^2 = \frac{D}{4a^2}$$

and its turning point is

$$(x, y) = (-\frac{b}{2a}, \frac{D}{4a^2})$$

Problems

1. Let x_1, x_2 be roots of equation $x^2 + 5x - 7 = 0$. Find:
   (a) $x_1^2 + x_2^2$
   (b) $(x_1 - x_2)^2$
   (c) $\frac{1}{x_1} + \frac{1}{x_2}$
   (d) $x_1^3 + x_2^3$
2. Sketch the graphs of the following functions and relations:
   (a) $y = x^2 - 3x$
   (b) $y = (x - 5)^2 - 10$

(c) $y = (x-3)^2 - 1$

(d) $y = x^2 - 4x - 8$

(e) $y = x^2 + x - 4$

3. Solve the following inequalities and sketch the graph.

   (a) $x^2 - 5x + 4 < 0$

   (b) $2x^2 + 5x - 3 > 0$

   (c) $x^2 > 1 + x$

   (d) $-x^2 + 2x - 4 > 0$

4. Of all the rectangles with perimeter 4, which one has the largest area? *Hint:* if sides of the rectangle are a and b, then the area is $A = ab$, and the perimeter is $2a + 2b = 4$. Thus $b = 2 - a$, so one can rewrite A using only a.