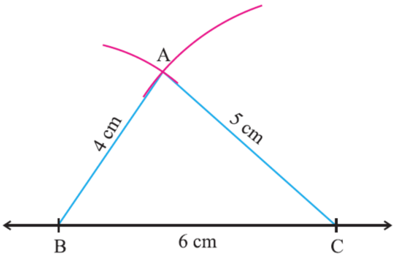
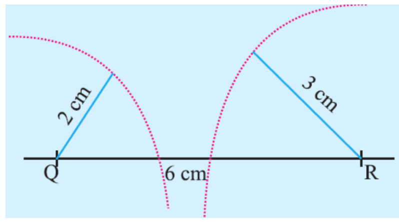
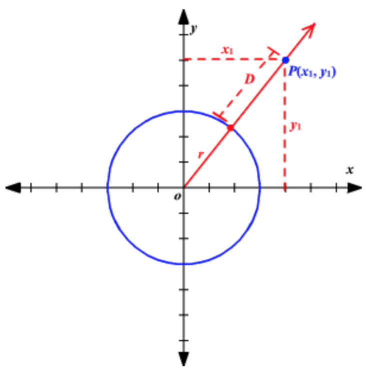
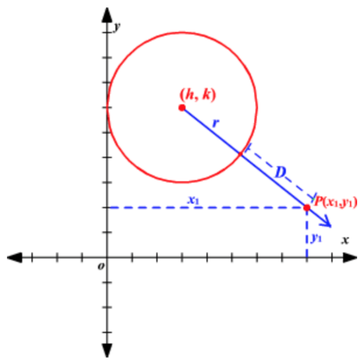



# GEOMETRY IN MATHKANGAROO

The triangle inequality is a very important mathematical relation. It is a core requirement for any metric, or distance function. While a proof for points in the plane that form a triangle is easy, the general case of the inequality assumes familiarity with more advanced concepts of the Euclidean space.

**Triangle Inequality Theorem**

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**The distance between a circle C and a point P**

The distance between a circle C with equation $x^2 + y^2 = r^2$, which is centered at the origin and a point P(x1,y1)?

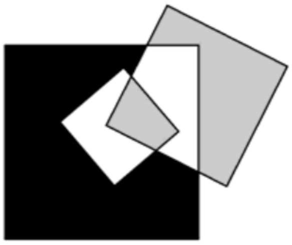
Using the triangle inequality, the shortest distance between the circle and an outside point P will be the difference of the distance of the point from the origin and the radius of the circle. Using the Distance Formula we obtain $|\sqrt{(x_0)^2 + (y_0)^2} - r|$

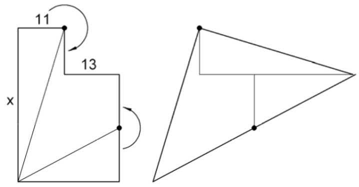
The formula works whether P is inside or outside the circle.

If the circle has a center in O(a,b) and a radius r, the shortest distance between the point $P(x, y)$ and the circle is $|\sqrt{(x_0 - a)^2 + (y_0 - b)^2} - r|$

**EXERCICE** 23/MathKang 2011 (Austria)

Each one of the three birds Isaak, Max and Oskar has its own nest. Isaak says: I am more than twice as far away from Max as I am from Oskar. Max says: I am more than twice as far away from Oskar as I am from Isaak. Oskar says: I am more than twice as far away from Max as I am from Isaak. At least two of them speak the truth. Who is lying? A) Isaak B) Max C) Oskar D) nobody E) It can not be decided from the information given.






Hints:

Fix a distance and use any known inequalities to find a contradiction with the triangle inequality that any distance has to respect in the Euclidian space.

What type of contradiction, ie. statements cannot co-exist. Those will be your "questioned" statements.

If you find at least one statement that seems correct. Analyze it. Each statement uses two distances. Denote one distance with an unknown and try to find limits on the other two distances in terms of this unknown. Compare them with the given "questioned" statements

**EXERCICE** 24/MathKang 2011 (Austria)

On the inside of a square with side length 7 cm another square is drawn with side length 3 cm. Then a third square with side length 5 cm is drawn so that it cuts the first two as shown in the picture on the right. How big is the difference between the black area and the grey area?

Denote all the different parts of the drawing and write all the possible relations: Ws: white area inside the small 3x3 square

Wm: white area inside the medium 5x5 square

Pi: pink inside the big square

Po: pink outside the big square

B: black area, inside the big square

Ws+Pi=9 ; Pi+Po+Wm=25; B+Ws+Pi+Wm=49

Hint: You are looking for B-Pi-Po . What sequence of operations between these three relations can give B-Pi-Po?

**EXERCICE** 29/MathKang 2011 (Austria) The figure on the left consists of two rectangles. Two side lengths are marked: 11 and 13. The figure is cut into three parts along the two lines drawn inside. These can be put together to make the triangle shown on the right. How long is the side marked x?

A) 36 B) 37 C) 38 D) 39 E) 40

Hint: Identify everything that you know before rotations take place, not only the given ones, but also what one can derive.

Identify after rotations where are the known parts and where are is the unknown.

**EXERCICE** 26/MathKang 2011 (Austria) In a convex quadrilateral ABCD with AB = AC, the following holds true: $m\angle BAD = 80$,$m\angle ABC = 75$, $m\angle ADC = 65$. How big is $m\angle BDC$ ? (Note: In a convex quadrilateral all internal angles are less than 180.)

A) 10 B) 15 C) 20 D) 30 E) 45

Hints: Find angle C, draw both diagonals and use $\triangle ABC$ isosceles to find all its angles. We need more relations in order to arrive at $m\angle BDC$, so try to see if by computing the angles more triangles become isosceles.

To find the measure of $\angle C$ one needs

**Theorem** The sum of the angles of a quadrilateral is 360.

Why is this true? Can you prove it?