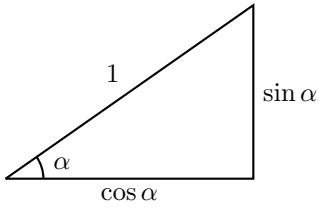
Basic trigonometry

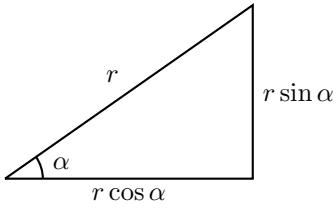
For any angle α, we define two numbers, $\sin\alpha$ (sine) and $\cos\alpha$ (cosine) as the lenghts of the legs in th right triangle with hypotenuse 1 and angle α:



In general, there is no simple formula for computing $\sin(\alpha)$ and $\cos(\alpha)$. However, there are some special angles, for which sin and cos can be computed explicitly:

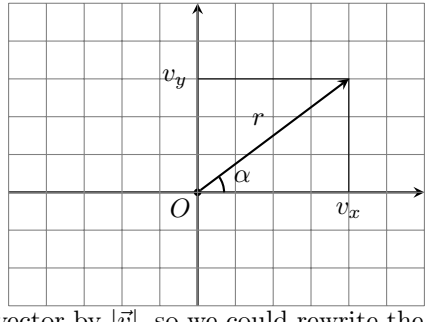
α	$\sin(\alpha)$	$\cos(\alpha)$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle α and hypotenuse r, then the sides will be $r\sin\alpha$ and $r\cos\alpha$:



In particular, this shows that if we have a vector $\vec{v}$ that has length r and forms angle α with the x axis, then its x and y components are

$$v_x = r\cos\alpha, \qquad v_y = r\sin\alpha$$



Note that it is also common to denote length of a vector by $|\vec{v}|$, so we could rewrite the previous formula by

$$v_x = |\vec{v}|\cos\alpha, \qquad v_y = |\vec{v}|\sin\alpha$$

Homework

In this homework, you can use the calculator to compute sin and cos of various angles.

1. Consider a regular hexagon. If we place a unit mass at each vertex, where would be the center of masses of the resulting system?

2. Vector $\vec{v}$ has length 1; vector $\vec{w}$ has length 2, and the angle between them is 30°. What is the length of vector $\vec{v} + \vec{w}$? [Hint: introduce a coordinate system so that $\vec{v}$ goes along the x-axis, and write coordinates of each of the vectors in this system.]

3. A ship travels for 3 miles north, then turns and goes for 2 miles northeast, then for another 5 miles north-northeast. Where will it be at the end? how far east and north of the original position? [Northeast means that its direction bisects the angle between north and east directions, thus forming an angle of 45° with due north. North-northeast means that this direction bisects the angle between north and north-east, thus forming 22.5° angle with due north.]

4. Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? [Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.]

5. Consider a parallelogram $ABCD$ with $AB = 1$, $AD = 3$, $\angle A = 40^\circ$. Find the lengths of diagonals in this parallelogram.

6. Prove that the area of a triangle $\triangle ABC$ can be computed using the formula $A = \frac{1}{2} AB \cdot AC \cdot \sin \angle A$. [Hint: what is the altitude from vertex B?]