You can solve an equation that contains a perfect square by finding square roots. The simplest of this type of equation has the form $ax^2 = c$.

Problem 1 Solving by Finding Square Roots

What is the solution of each equation?

A $4x^2 + 10 = 46$

$4x^2 = 36$ Rewrite in $ax^2 = c$ form.

$\frac{4x^2}{4} = \frac{36}{4}$ Isolate x^2.

$x^2 = 9$

$x = \pm 3$ Find square roots.

B $3x^2 - 5 = 25$

$3x^2 = 30$ Rewrite in $ax^2 = c$ form.

$\frac{3x^2}{3} = \frac{30}{3}$ Isolate x^2.

$x^2 = 10$

$x = \pm\sqrt{10}$ Find square roots.

Sometimes an equation shows a perfect square trinomial equal to a constant. To solve, factor the perfect square trinomial into the square of a binomial. Then find square roots.

Problem 3 Solving a Perfect Square Trinomial Equation

What is the solution of $x^2 + 4x + 4 = 25$?

THINK	WRITE
Factor the perfect square trinomial.	$x^2 + 4x + 4 = 25$ $(x + 2)^2 = 25$
Find square roots.	$x + 2 = \pm 5$
Rewrite as two equations.	$x + 2 = 5$ or $x + 2 = -5$
Solve for x.	$x = 3$ or $x = -7$

TAKE NOTE Key Concept

Completing the Square

You can form a perfect square trinomial from $x^2 + bx$ by adding $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Problem 4 Completing the Square

What value completes the square for $x^2 - 10x$? Justify your answer.

$x^2 - 10x$	Identify b; $b = -10$.
$\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$	Find $\left(\frac{b}{2}\right)^2$.
$x^2 - 10x + 25$	Add the value of $\left(\frac{b}{2}\right)^2$ to complete the square.
$x^2 - 10x + 25 = (x - 5)^2$	Rewrite as the square of a binomial.

By completing the square, you can solve quadratic equations that cannot be solved by factoring.

TAKE NOTE Key Concept

Solving an Equation by Completing the Square

1. Rewrite the equation in the form $x^2 + bx = c$. To do this, get all terms with the variable on one side of the equation and the constant on the other side. Divide all the terms of the equation by the coefficient of x^2 if it is not 1.
2. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to each side of the equation.
3. Factor the trinomial.
4. Find square roots.
5. Solve for x.

Problem 5 Solving by Completing the Square

What is the solution of $3x^2 - 12x + 6 = 0$?

$3x^2 - 12x + 6 = 0$

$3x^2 - 12x = -6$ Rewrite. Get all terms with x on one side of the equation.

$\frac{3x^2}{3} - \frac{12x}{3} = \frac{-6}{3}$ Divide each side by 3 so the coefficient of x^2 will be 1.

$x^2 - 4x = -2$ Simplify.

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$ Find $\left(\frac{b}{2}\right)^2$.

$x^2 - 4x + 4 = -2 + 4$ Add 4 to each side.

$(x - 2)^2 = 2$ Factor the trinomial.

$x - 2 = \pm\sqrt{2}$ Find square roots.

$x = 2 \pm \sqrt{2}$ Solve for x.

graph of a **quadratic function**, which you can write in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. You can also write quadratic functions in other forms.

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

The **axis of symmetry** is a line that divides the graph of the function (a parabola) into two mirror images. The equation of the axis of symmetry is $x = h$.

The **vertex of the parabola** is (h, k), the intersection of the parabola and its axis of symmetry.

You can complete the square to rewrite a quadratic function in vertex form. As shown in Problem 6, you can choose to add $\left(\frac{b}{2}\right)^2$ to one side of the equation and then subtract it from the same side, instead of adding $\left(\frac{b}{2}\right)^2$ to each side of the equation.

Problem 6 Writing in Vertex Form

What is $y = x^2 + 4x - 6$ in vertex form? Name the vertex and y-intercept.

$y = x^2 + 4x - 6$

$y = x^2 + 4x + 2^2 - 6 - 2^2$ Add $\left(\frac{4}{2}\right)^2 = 2^2$ to complete the square. Also, subtract 2^2 to leave the function unchanged.

$y = (x + 2)^2 - 6 - 2^2$ Factor the perfect square trinomial.

$y = (x + 2)^2 - 10$ Simplify.

The vertex is $(-2, -10)$. The y-intercept is $(0, -6)$.

A • Practice

Solve each equation by finding square roots.
SEE PROBLEM 1.

12. $5x^2 = 80$

13. $x^2 - 4 = 0$

14. $2x^2 = 32$

15. $9x^2 = 25$

16. $3x^2 - 15 = 0$

17. $5x^2 - 40 = 0$

18. **Fitness** A rectangular swimming pool is 6 ft deep. One side of the pool is 2.5 times longer than the other. The amount of water needed to fill the swimming pool is 2160 cubic feet. Find the dimensions of the pool. SEE PROBLEM 2.

Solve each equation. SEE PR

19. $x^2 + 6x + 9 = 1$

20. $x^2 - 4x + 4 = 100$

21. $x^2 - 2x + 1 = 4$

22. $x^2 + 8x + 16 = \frac{16}{9}$

23. $4x^2 + 4x + 1 = 49$

24. $x^2 - 12x + 36 = 25$

25. $25x^2 + 10x + 1 = 9$

26. $x^2 - 30x + 225 = 400$

27. $9x^2 + 24x + 16 = 36$

Complete the square. SEE PROBLEM 4.

28. $x^2 + 18x + \square$

29. $x^2 - x + \square$

30. $x^2 - 24x + \square$

31. $x^2 + 20x + \square$

32. $m^2 - 3m + \square$

33. $x^2 + 4x + \square$

Solve each quadratic equation by completing the square. SEE PROBLEM 5.

34. $x^2 + 6x - 3 = 0$

35. $x^2 - 12x + 7 = 0$

36. $x^2 + 4x + 2 = 0$

37. $x^2 - 2x = 5$

38. $x^2 + 8x = 11$

39. $x^2 + 12 = 10x$

40. $x^2 - 3x = x - 1$

41. $x^2 + 2 = 6x + 4$

42. $2x^2 + 2x - 5 = x^2$

43. $4x^2 + 10x - 3 = 0$

44. $9x^2 - 12x - 2 = 0$

45. $25x^2 + 30x = 12$

Rewrite each equation in vertex form. SE

46. $y = x^2 + 4x + 1$

47. $y = 2x^2 - 8x + 1$

48. $y = -x^2 - 2x + 3$

49. $y = x^2 + 4x - 7$

50. $y = 2x^2 - 6x - 1$

51. $y = -x^2 + 4x - 1$

Find the value of *k* that would make the left side of each equation a perfect square trinomial.

53. $x^2 + kx + 25 = 0$

54. $x^2 - kx + 100 = 0$

55. $x^2 - kx + 121 = 0$

56. $x^2 + kx + 64 = 0$

57. $x^2 - kx + 81 = 0$

58. $25x^2 - kx + 1 = 0$

59. $x^2 + kx + \frac{1}{4} = 0$

60. $9x^2 - kx + 4 = 0$

61. $36x^2 - kx + 49 = 0$

Solve each quadratic equation by completing the square.

63. $x^2 + 5x - 3 = 0$

64. $x^2 + 3x = 2$

65. $x^2 - x = 5$

66. $x^2 + x - 1 = 0$

67. $3x^2 - 4x = 2$

68. $5x^2 - x = 4$

69. $x^2 + \frac{3}{4}x = \frac{1}{2}$

70. $2x^2 - \frac{1}{2}x = \frac{1}{8}$

71. $3x^2 + x = \frac{2}{3}$

72. $-x^2 + 2x + 4 = 0$

73. $-x^2 - 6x = 2$

74. $-0.25x^2 - 0.6x + 0.3 = 0$