

## MATH 7: ALGEBRAIC EXPRESSIONS AND ROOTS

### MATERIAL COVERED TODAY

Today we discussed how one works with algebraic expressions, i.e. expressions containing variables, such as  $2(x + 1) - 3$ . In particular, we discussed the following useful formulas:

1.  $(ab)^n = a^n b^n$
2.  $\sqrt{ab} = \sqrt{a}\sqrt{b}$
3.  $(a + b)^2 = a^2 + 2ab + b^2$
4.  $(a - b)^2 = a^2 - 2ab + b^2$
5.  $a^2 - b^2 = (a - b)(a + b)$

Replacing in the last equality  $a$  by  $\sqrt{a}$ ,  $b$  by  $\sqrt{b}$ , we get

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

which is very helpful in simplifying expressions with roots, for example:

$$\frac{1}{\sqrt{2} + 1} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

We also discussed solving simple equations: linear equation (i.e., equation of the form  $ax + b = 0$ , with  $a, b$  some numbers, and  $x$  the unknown) and equation where the left hand side is factored as product of linear factors, such as  $(x - 2)(x + 3) = 0$ .

### HOMEWORK

1. Without a calculator, compute

$$199999 \cdot 200001$$

Is there a shorter way of doing it than the straightforward multiplication?

2. Simplify the following expressions, writing them in the form  $\frac{f}{g}$ , where  $f, g$  are polynomials.

$$(a) \frac{1}{x+1} - \frac{1}{x-1} \quad (b) \left(1 + \frac{1}{x}\right) \div (x+1) \quad (c) \left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$

3. Factor (i.e., write as a product) the following expressions:

- |   |   |
|---|---|
| (a) $3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$ | (h) $4x^2 + 8xy + 4y^2$                       |
| (b) $a^2 - b^2 - 10b - 25$                      | (i) $(x - 2)^2 - 10(x - 2) + 25$ (typo fixed) |
| (c) $x^4 + 4$ (typo fixed)                      | (j) $a^2 + 4ab + 4b^2$                        |
| (d) $256 - a^8b^8$ (question updated)           | (k) $a^2 - 2a + 1$                            |
| (e) $\frac{1}{9}x^2 - 25$                       | (l) $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$ .]     |
| (f) $a^9 - 27$ (Skip this for now!)             | (m) $x^2 - 7$ [Hint: $7 = (\sqrt{7})^2$ .]    |
| (g) $(x - 2)^2 - (y + 3)^2$                     |   |

4. Write each of the following expressions in the form  $a + b\sqrt{3}$ , with rational  $a, b$ :

$$(a) (1 + \sqrt{3})^2 \quad (b) (1 + \sqrt{3})^3$$
$$(c) \frac{1}{1 - 2\sqrt{3}} \quad (d) \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \quad (e) \frac{1 + 2\sqrt{3}}{\sqrt{3}}$$

5. Solve the equation  $(x - 1)^2 = 6$ .

6. Solve the following equations. Carefully write all the steps in your argument. Please do not use calculators.

- |                            |                                 |                    |
|----------------------------|---------------------------------|--------------------|
| (a) $(x^2 - 1)(x + 2) = 0$ | (d) $(x - 3)(x + 4) = 0$        | (f) $x^2 + 4x = 0$ |
| (b) $\frac{x+2}{x+3} = 2$  | (e) $\frac{x^2-4}{x+1} = x - 2$ | (g) $x^3 + 4x = 0$ |
| (c) $5(x + 1) = 3x + 2$    |                                 |                    |