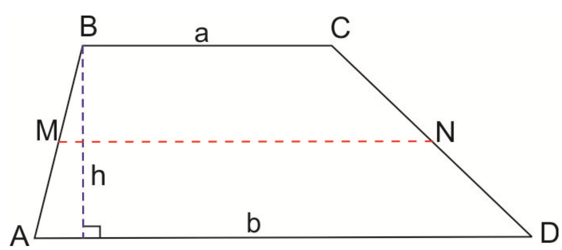
Homework for October 1, 2017.

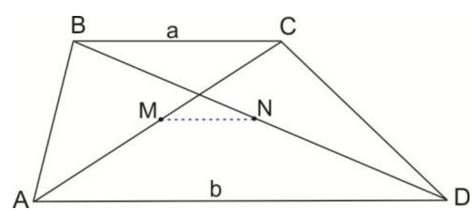
Geometry.

Review the classwork handout. Please pay attention to the alternative heuristic proofs of the Thales proportionality theorem that was skipped in class. Skip the problems that were solved in class.

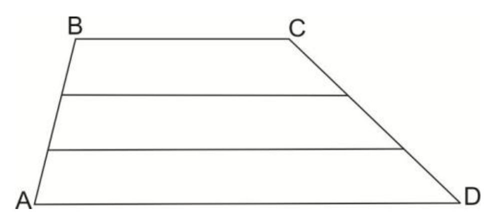
Problems.

A *trapezoid* is a quadrilateral with two parallel sides. An *isosceles trapezoid* is a trapezoid in which base angles are equal and therefore the lengths of the left and the right side are also equal. It is clear that the area of a trapezoid is $S = h\frac{a+b}{2}$.





1. Prove that segment connecting midpoints of the diagonals of a trapezoid is parallel to its bases and that the length of this segment is half of the difference of bases, $|MN| = \frac{b-a}{2}$.



2. Two lines parallel to trapezoid's bases divide sides into segments of equal length. Bases of the trapezoid are 2 and 5 sm. What are the lengths of other parallel segments?
3. Given two segments of lengths a and b, construct,
   a. A segment of length a^2
   b. A segment of length 1/a
   c. A segment of length ab
   d. A segment of length a/b

4. Using a ruler and a compass, construct a triangle given the midpoints of its three sides.

The words “construct...” mean “describe how this can be constructed using ruler and compass". In these problems, you can freely reuse the constructions which were done before - e.g., you can just say “construct the perpendicular from this point to this line", without repeating how it can be done - you have already discussed it once, so there is no need to repeat it.

In all the problems, you are only allowed to use theorems we had proved before!

Algebra.

Review the classwork handout. Solve the following problems.

1. Let P, Q and R be some logical propositions. Write a negation of
   a. $((P \wedge Q) \vee R)$
   b. $(P \wedge (Q \vee R))$
   c. $(P \Rightarrow (Q \vee R))$
   d. $(P \Rightarrow (Q \Leftrightarrow R))$
   e. $(P \Leftrightarrow (Q \Rightarrow R))$
2. Solve the following equation, writing your solution with properly tracking the equivalence of the transformed forms,

$$\frac{x^2-3x+2}{x}+\frac{x}{x^2-3x+2}=2.5$$

3. Prove that

$$\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{b}=\frac{a-c}{b-d}=\frac{a+c}{b+d}.$$

Consider how this property is used in proving the Thales theorem.

4. Factorize expressions
   a. $1+a+a^2+a^3+a^4+a^5$
   b. $1+a-2a^2-2a^3+a^4+a^5$
5. Simplify expressions:
   a. $\frac{x+y}{x}-\frac{x}{x-y}+\frac{y^2}{x^2-xy}$
   b. $\frac{\sqrt{1+\left(\frac{x^2-1}{2x}\right)^2}}{(x^2+1)\frac{1}{x}}$
   c. $\frac{\sqrt{2b+2\sqrt{b^2-4}}}{\sqrt{b^2-4}+b+2}$
6. Find all natural numbers a and b, such that $a^3-b^3=19$
7. *Find the roots of the following equation, writing your solution with properly tracking the equivalence of the transformed forms,

$$\frac{14}{20-6x-2x^2}+\frac{x^2+4x}{x^2+5x}-\frac{x+3}{x-2}+3=0.$$