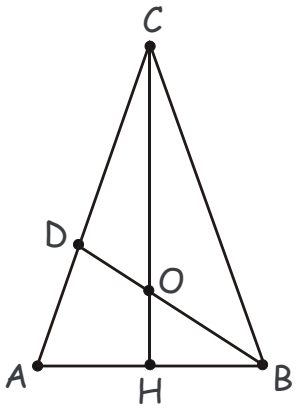
Homework for November 5, 2017.

Geometry.

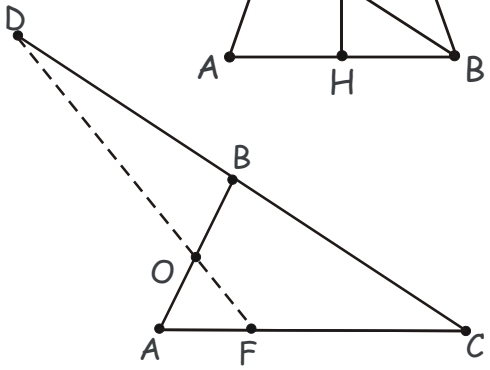
Review the classwork handout. Solve the unsolved problems from previous homeworks. Try solving the following problems from the previous homework using the method of point masses and the Law of Lever.

Problems.

1. Prove that if a polygon has several axes of symmetry, they are all concurrent (cross at the same point).
2. Prove that medians of a triangle divide one another in the ratio 2:1, in other words, the medians of a triangle "trisect" one another (Coxeter, Gretzer, p.8).
3. In isosceles triangle ABC point D divides the side AC into segments such that |AD|:|CD|=1:2. If CH is the altitude of the triangle and point O is the intersection of CH and BD, find the ratio |OH| to |CH|.



4. Point D belongs to the continuation of side CB of the triangle ABC such that |BD| = |BC|. Point F belongs to side AC, and |FC| = 3|AF|. Segment DF intercepts side AB at point O. Find the ratio |AO|:|OB|.



Algebra.

Review the classwork handout and complete the exercises. Solve the remaining problems from the previous homework (you may skip the ones considered in class). Solve the following problems.

1. Find the following sum.

$$\left(2+\frac{1}{2}\right)^2+\left(4+\frac{1}{4}\right)^2+\cdots+\left(2^n+\frac{1}{2^n}\right)^2$$

2. The lengths of the sides of a triangle are three consecutive terms of the geometric series. Is the common ratio of this series, q, larger or smaller than 2?
3. Solve the following equation,

$\frac{x-1}{x}+\frac{x-2}{x}+\frac{x-3}{x}+\cdots+\frac{1}{x}=3$, where x is a positive integer.

4. Find the following sum,
   a. $1+2\cdot 3+3\cdot 7+\cdots+n\cdot(2^n-1)$
   b. $1\cdot 3+3\cdot 9+5\cdot 27+\cdots+(2n-1)\cdot 3^n$
5. Numbers $a_1, a_2, \ldots, a_n$ are the consecutive terms of a geometric progression, and the sum of its first n terms is S_n. Show that,

$$S_n=a_1a_n\left(\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right)$$

6. Prove that three terms shown below are the three terms of the geometric progression, and find the sum of its first n terms, beginning with the first one below,

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{1}{3-\sqrt{3}}+\frac{1}{6}+\cdots$$

7. What is the maximum value of the expression, $(1+x)^{36}+(1-x)^{36}$ in the interval $|x|\leq 1$?
8. Find the coefficient multiplying x^9 after all parenthesis are expanded in the expression, $(1+x)^9+(1+x)^{10}+\cdots+(1+x)^{19}$.