November 19, 2017

Geometry.

The Inscribed Angle Theorem.

Theorem. An angle α inscribed in a circle is half of the central angle $\beta = 2\alpha$ that subtends the same arc on the circle (Fig.1), or complete half of it to 180.
Corollary. The angle does not change as its apex is moved to different positions on the circle.

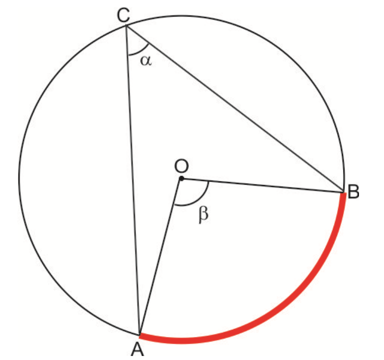

Fig. 1

Proof. First, let us deal with the simple case when one of the rays of angle ACB' passes through the center of the circle (Fig. 2). $\angle AOB'(\beta)$ is a central angle that subtends the same arc as $\angle ACB'$ (α). Triangle AOC is an isosceles triangle because $|OA| = |OC|$, so angle $\angle OAC$ and angle $\angle OCA$ are equal and angle $\angle AOC = 180 - 2\alpha$, but it is also equal $180 - \beta$ as a supplement angel to angle β.

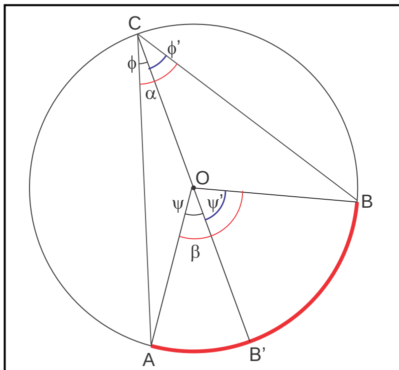

Fig. 2

$$\angle AOC = 180 - 2\alpha = 180 - \beta \Rightarrow \beta = 2\alpha.$$

In the case when center of the circle placed inside of angle ACB we can divide the angle ACB with a ray CB' passing through the center of the circle (Fig. 3). Now we have two inscribed angles: angle ACB' and angle $B'CB$, each of them has one side which passes through the center of the circle and can use previous part to proof that $\beta = 2\alpha$.

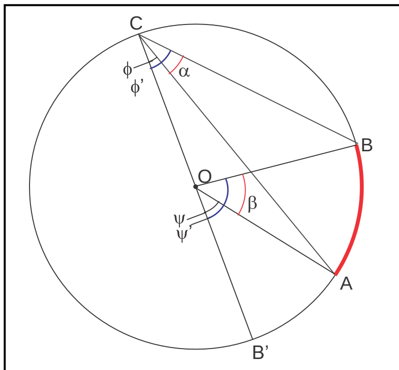

Fig. 3

$$\alpha = \phi + \phi',$$

$$\beta = \psi + \psi' = 2\phi + 2\phi' = 2(\phi + \phi') = 2\alpha.$$

When center of the circle is outside of inscribed angle, we can draw a ray from a vertex of our angle through the center the circle (Fig. 4). Then the angle $\angle ACB(\alpha) = \angle B'CB(\phi') \; - \; \angle B'CA(\phi)$ and we again can use the first part.

$$\beta \; = \; \psi' - \; \psi = \; 2\phi' - \; 2\phi = \; 2(\phi' \; - \; \phi) \; = \; 2\alpha.$$

Only the case of obtuse angle is left. In this case the ray CB' passes through the center of the circle and divides angle ∠ACB into two angles ϕ and ϕ' They are not now half of the angles ψ and ψ', but half of their supplement angles χ and χ' therefore,

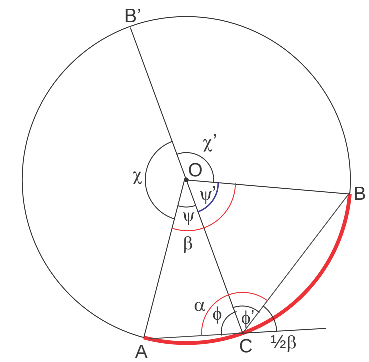

Fig. 4

$$\alpha = \frac{1}{2}\chi + \frac{1}{2}\chi' = \frac{1}{2}(\chi + \chi') = \frac{1}{2}(180 - \psi + 180 - \psi') = 180 - \frac{1}{2}(\psi + \psi') = 180 - \frac{1}{2}\beta.$$

The Rowland circle.

In scientific diffraction instruments it is often desirable to have a diffraction mirror shaped in a way such that the reflection of a beam of light, or particles, emanating from a point source, and focused to a point, corresponds to the same angle between the incident and the reflected (diffracted) beam for any point on the mirror. Such mirror is a segment of the so-called Rowland circle.