March 11, 2017

Math battle 9.

1. How many natural numbers <1000 are not divisible by 11, 13 and 17?
2. Four friends, A., B., C. and D., decided to exchange presents. They agreed that each one prepares a present, which will then be randomly drawn. Hence, each can get, with equal probability, any of the four presents. What is the probability that no one gets his/her own present, while A. gets the present from D.?
3. 10 students are solving problems of a math competition. Each of 10 problems was solved by the same number of students, but no two students have solved the same number of problems. One of the students, Max, has solved problems 1 through 5 and has not solved problems 6 through 9. Has Max solved problem 10?

4. Solve the following equation: $2018 \ fractions \begin{cases} \cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{\cdots}}} \\ \qquad\qquad -\cfrac{1}{1-\cfrac{1}{1-\frac{1}{1-x}}} \end{cases} = \dfrac{1}{2018}$

5. Is it possible to put in each vertex of any given triangle ABC a number such that for every edge, its length is equal to the sum of numbers at its endpoints?
6. Prove that the sum of internal angles of an n-gon (a polygon with n vertices) is $(n-2)\cdot 180°$.
7. Let ABC be a right triangle, with $\angle A = 90°$, and $K \in BC$ be such that $AB = AK$. If we know that segment AK bisects the angle bisector CL, what are the angles of ΔABC?
8. What is the largest combined volume of two spherical balls that can fit in a cubic box with side 1?
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