Homework for April 1, 2018.

Algebra/Trigonometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on trigonometric functions. Additional reading on trigonometric functions is Read Gelfand & Saul, Trigonometry, Chapter 2 (pp. 42-54) and Chapters 6-8 (pp. 123-163),
http://en.wikipedia.org/wiki/Trigonometric_functions
http://en.wikipedia.org/wiki/Sine. Solve the following problems.

Problems.

1. Find the sum of the following series,

$$S = \cos x + \cos 2x + \cos 3x + \cos 4x + \cdots + \cos Nx$$

(hint: multiply the sum by $2\sin x/2$)

2. Find all x for which,
   a. $\sin x \cos x = \frac{1}{2}$
   b. $\sin x \cos x = \frac{\sqrt{3}}{2}$
3. Simplify the following expression:
   a. $(1 + \sin\alpha)(1 - \sin\alpha)$
   b. $(1 + \cos\alpha)(1 - \cos\alpha)$
   c. $\sin^4\alpha - \cos^4\alpha$
   d. $\cos^2\left(\alpha - \frac{\pi}{6}\right) + \cos^2\left(\alpha + \frac{\pi}{6}\right) + \sin^2\alpha =$
4. Calculate:
   a. $\cos 75° + \cos 15° =$
   b. $\cos\frac{\pi}{12} - \cos\frac{5\pi}{12} =$
5. Solve the following equation:
   a. $\cos^2\pi x + 4\sin\pi x + 4 = 0$

6. Prove the following equalities:
   a. $\frac{1}{\sin\alpha}+\frac{1}{\tan\alpha}=\cot\frac{\alpha}{2}$
   b. $\sin^2\left(\frac{7\pi}{8}-2\alpha\right)-\sin^2\left(\frac{9\pi}{8}-2\alpha\right)=\frac{\sin 4\alpha}{\sqrt{2}}$
   c. $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=4\sin^2\frac{\alpha-\beta}{2}$
   d. $\frac{\cot^2 2\alpha-1}{2\cot 2\alpha}-\cos 8\alpha\cot 4\alpha=\sin 8\alpha$
   e. $\sin^6\alpha+\cos^6\alpha+3\sin^2\alpha\cos^2\alpha=1$
   f. $\frac{\sin 6\alpha+\sin 7\alpha+\sin 8\alpha+\sin 9\alpha}{\cos 6\alpha+\cos 7\alpha+\cos 8\alpha+\cos 9\alpha}=\tan\frac{15\alpha}{2}$
   g. $\sin^6\alpha+\cos^6\alpha=\frac{5+3\cos 4\alpha}{8}$
   h. $16\sin^5\alpha-20\sin^3\alpha+5\sin\alpha=\sin 5\alpha$
   i. $\frac{\cos 64°\cos 4°-\cos 86°\cos 26°}{\cos 71°\cos 41°-\cos 49°\cos 19°}$
   j. $\sin 20°\sin 40°\sin 60°\sin 80°=\frac{3}{16}$
   k. $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}=4$
7. Simplify the following expressions:
   a. $\sin^2\left(\frac{\alpha}{2}+2\beta\right)-\sin^2\left(\frac{\alpha}{2}-2\beta\right)$
   b. $2\cos^2 3\alpha+\sqrt{3}\sin 6\alpha-1$
   c. $\cos^4 2\alpha-6\cos^2 2\alpha\sin^2 2\alpha+\sin^4 2\alpha$
   d. $\sin^2(135°-2\alpha)-\sin^2(210°-2\alpha)-\sin^2 195°\cos(165°-4\alpha)$
   e. $\frac{\cos 2\alpha-\cos 6\alpha+\cos 10\alpha-\cos 14\alpha}{\sin 2\alpha+\sin 6\alpha+\sin 10\alpha+\sin 14\alpha}$
8. Find the period of the function y $=\sin 5x-2\sin 7x$
9. Let A, B and C be angles of a triangle. Prove that

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$$

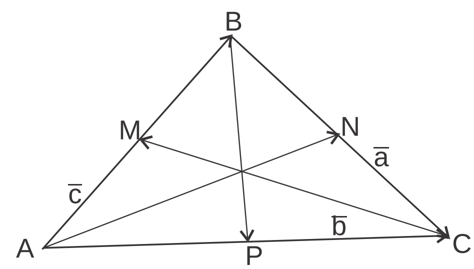
10. Solve the following equations and inequalities:

f. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$
g. $\cos 3x - \sin x = \sqrt{3}(\cos x - \sin 3x)$
h. $\sin^2 x - 2\sin x \cos x = 3\cos^2 x$
i. $\sin 6x + 2 = 2\cos 4x$
j. $\cot x - \tan x = \sin x + \cos x$
k. $\sin x \geq \pi/2$
l. $\sin x \leq \cos x$

Geometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on vectors. Solve the following problems.

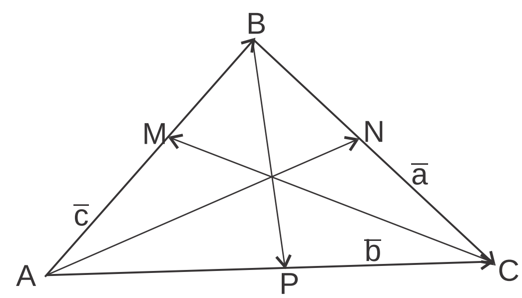
Problems.



1. In a triangle ABC, vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ (**c**, **b** and **a**) are the sides. $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ are the medians.

   a. Express vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ through vectors **c**, **b** and **a.**

   b. Find the sum of vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$**.**



2. Solve the same problem for bisectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ in a triangle ABC.

3. Coxeter, Greitzer, problem #9 to Sec. 2.1 (p. 31): How far away is the horizon as seen from the top of a mountain 1 mile high? (Assume the Earth to be a sphere of diameter 7920 miles.)

4. In a rectangle $ABCD$, A_1, B_1, C_1 and D_1 are the mid-points of sides AB, CD, BC and DA, respectively. M is the crossing point of the segments A_1B_1, and C_1D_1, connecting two pairs of midpoints.

   a. Express vector $\overrightarrow{A_1M}$ through $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$.

   b. Prove that M is the mid-point of segments, A_1B_1 and C_1D_1, i.e. $|A_1M| = |MB_1|$ and $|C_1M| = |MD_1|$.

5. In a parallelogram $ABCD$, find $\overrightarrow{AB} + \overrightarrow{BD} - 2\overrightarrow{AD}$.

6. M is a crossing point of the medians in a triangle ABC. Prove that $\overrightarrow{AM} = \frac{1}{3}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$.

7. For three points, $A(-1,3)$, $B(2,-5)$ and $C(3,4)$, find the (coordinates of) following vectors,

   a. $\overrightarrow{AB} - \overrightarrow{BC}$

   b. $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC}$

   c. $\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA}$

8. For two triangles, ABC and $A_1B_1C_1$, $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$. Prove that medians of these two triangles cross at the same point M.