Homework for May 6, 2018.

Algebra/Geometry. Complex numbers.

Review the classwork handout on complex numbers. Please, complete the problems from the previous homework assignments, some of which are repeated below. Solve the following problems.

Problems.

1. Using the de Moivre formula, prove the following equalities:
   a. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
   b. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$
   c. $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$
   d. $\sin 4\alpha = 4\sin\alpha\cos^3\alpha - 4\cos\alpha\sin^3\alpha$
   e. $\sin 5\alpha = 16\sin^5\alpha - 20\sin^3\alpha + 5\sin\alpha$
   f. $\cos 5\alpha = \cdots$ (find the expression)
2. (i) find the magnitude and the argument, (ii) compute the inverse, and (iii) find the magnitude and the argument of the inverse for the following complex numbers:
   a. $1 + i$
   b. $-i$
   c. $1 + ix$
   d. $\frac{\sqrt{3}}{2} + \frac{i}{2}$
   e. $\frac{1}{2-i} - \frac{1}{2+i}$
3. Compute and write in the trigonometric form:
   a. $(1 + i)^8$
   b. $(1 - i)^{10}$
   c. $(1 - i)^{-10}$
   d. $(3 + 4i)^{-1}$
   e. $\left(i\sqrt{3} - 1\right)^{17}$
   f. $\left(\frac{1-i}{\sqrt{2}}\right)^5$
   g. $\left(\frac{1+i}{1-i}\right)^{2015}$

4. Find a complex number z whose magnitude is 2 and the argument $Arg(z) = \frac{\pi}{4} = 45°$.
5. Draw the following sets of points on complex plane.
   a. $\{z|Re(z) = 1\}$
   b. $\left\{z|Arg(z) = \frac{3\pi}{4} = 135°\right\}$
   c. $\{z|\ |z| = 1\}$
   d. $\{z|Re(z^2) = 0\}$
   e. $\{z|\ |z^2| = 2\}$
   f. $\{z|\ |z-1| = 1\}$
   g. $\{z|\ z + \bar{z} = 1\}$
6. Prove that for any complex number z, we have
   a. $|\bar{z}|=|z|$, $Arg(\bar{z}) = -Arg(z)$
   b. $\frac{\bar{z}}{z}$ has magnitude 1; check this for $z = 1 - i$.
7. If z has magnitude 2 and argument $\frac{\pi}{2}$ and w has magnitude 3 and argument $\frac{\pi}{3}$, what will be the magnitude and the argument of zw? Write it in the form $a + bi$.
8. Let $P(x)$ be a polynomial wit real coefficients.
   a. Prove that for any complex number z, we have $\overline{P(z)} = P(\bar{z})$
   b. Let z be a complex root of this polynomial, $P(z) = 0$. Prove that then $\bar{z}$ is also a root, $P(\bar{z}) = 0$.
9. Solve the equation $x^3 - 4x^2 + 6x - 4 = 0$. Find the sum and product of the roots in two ways: by using Vieta formulas and by explicit computation. Check that the results match.

Algebra: polynomials recap.

Review the classwork notes on polynomials, factorization and Vieta theorem. Solve the following problems.

Problems.

1. Write Vieta formulae for the cubic equation, $x^3 + Px^2 + Qx + R = 0$. Let x_1, x_2 and x_3 be the roots of this equation. Find the following combination in terms of P, Q and R,
   a. $(x_1 + x_2 + x_3)^2$
   b. ${x_1}^2 + {x_2}^2 + {x_3}^2$
   c. $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}$
   d. $(x_1 + x_2 + x_3)^3$
2. The three real numbers x, y, z, satisfy the equations

$$x + y + z = 6$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{6}$$

$$xy + yz + zx = 11$$

   a. Find a cubic polynomial whose roots are x, y, z
   b. Find x, y, z
3. Find two numbers u, v such that

$$u + v = 6$$

$$uv = 13$$

4. Find three numbers, a, b, c, such that

$$a + b + c = 2$$

$$ab + bc + ca = -7$$

$$abc = -14$$

5. Find all real roots of the following polynomial and factor it.

a. $x^8 + x^4 + 1$
b. $x^4 - x^3 + 5x^2 - x - 6$
c. $x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$

6. Perform the long division, finding the quotient and the remainder, on the following polynomials.
a. $(x^3 - 3x^2 + 4) \div (x^2 + 1)$
b. $(x^3 - 3x^2 + 4) \div (x^2 - 1)$