Acceleration.

Last time we defined "force": Force is the reason of change of momentum. It is that coefficient in the equation for momentum change:

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

(remember how we mentioned that the longer is time Δt of action, the larger is the change…)

But in examples that we considered the "change of momentum" means really "change of velocity". Suppose at some moment a force acts on a body. Suppose mass of the body stays the same. The change of momentum is:

$$\Delta\vec{p} = \vec{p}_{after} - \vec{p}_{before} = m\vec{v}_{after} - m\vec{v}_{before} = m\left(\vec{v}_{after} - \vec{v}_{before}\right) = m\Delta\vec{v}$$

Thus

$$\vec{F}\,\Delta t = m\Delta\vec{v}$$

Or

$$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t}$$

So when mass stays the same our *definition* of force says that

$$\vec{F} = m\vec{a}$$

where the quantity

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

has meaning of the velocity of change of the velocity. It is called ***acceleration***.

The statement

$$\vec{F} = m\vec{a}$$

is called "the second Newton's law". We introduced it as definition of force, but the fact that we could introduce this definition in the first formula above is a fundamental property of Nature which we discovered experimentally. So it is not just a definition – it is a law of physics which claims that almost all interactions around us can be encoded as reasons for the acceleration, changing of the speed over (meaning not only "divided by" but also "during") time.

I could envision some other mysterious Universe where the interactions cause change of coordinate – but when interaction is zero everything stops moving. That would be a boring Universe – cold and motionless, like in the Snow Queen's kingdom, even worse - a Universe without inertia… or I could envision that in the other Universe interaction causes change of acceleration, not velocity – so acceleration changes when forces act and everything accelerates with the same acceleration when forces disappear. This would be a crazy Universe where you are not left alone even when you are left alone….In these two imaginary Universes many things intuitively obvious for us would look very different …. Well, luckily we live in our Universe, where it seems that Newton's law works and forces cause acceleration, not anything else…

It is usually better to think about Newton's law this way:

$$\vec{a} = \frac{\vec{F}}{m}$$

So the acceleration is the result of applying the force.

Forces around us. Gravity.

Last time we discussed the gravity force:

$$F = G\frac{Mm}{R^2}$$

and we discussed that close to the Earth surface this implies

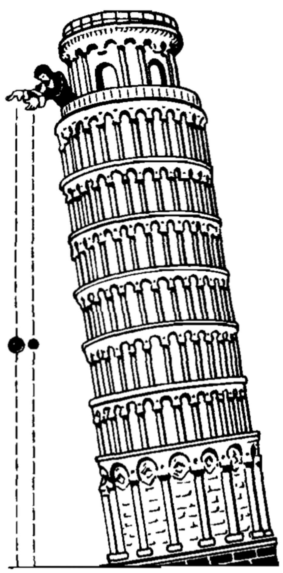
$$F = mg$$

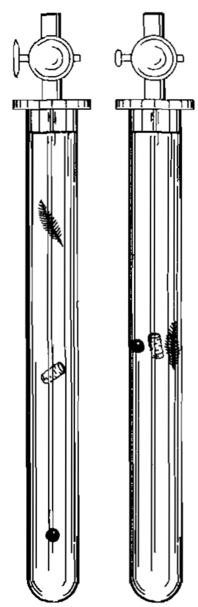
$$g = G\frac{M_{earth}}{R_{earth}^2} \approx 9.8\,m/s^2$$

Compare this to the second Newton's law:

$$a = \frac{F}{m} = g$$

so this g has the meaning of acceleration of a falling body. Note how it has no dependence on mass !





In the picture on the very right, when you remove the air from the test tube, the feather, the cork and the rock fall together, while when you have air it would slow down the lighter objects.

Forces around us. Hooke's law.

Suppose some quantity $f(x)$ depends on x and you know it in a particular point $x = x_0$. Here x is not necessarily the coordinate – this may be any physical characteristic – volume, pressure of a gas, angle of the string of the pendulum etc. Suppose you change that x_0 to $x_0 + \Delta x$ where this Δx is very small (compared to some typical value of this quantity, for example change of length of the spring is very small compared to the string itself). In many cases (in most cases when graph of $f(x)$ looks smooth and "reasonable") one may write the change of that $f(x)$ as

$$f(x_0 + \Delta x) - f(x_0) = A_1\,\Delta x + A_2(\,\Delta x)^2 + A_3(\,\Delta x)^3 + A_4(\,\Delta x)^4 + \cdots$$

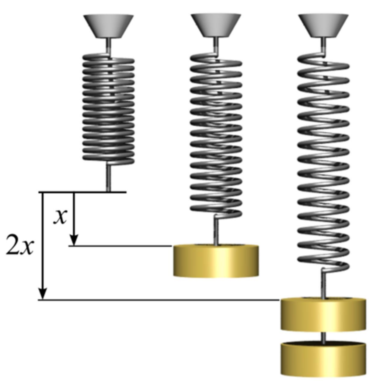
where $A_1, A_{2,}A_3. A_{4,\dots}$ are some numbers which can be computed for a given value of x_0.

This formula is called Taylor Series and we are definitely not trying to prove the validity of it, but just note that every next term is smaller than the previous one – really, $(\,\Delta x)^2$ is smaller than Δx for small Δx and so on. This is natural – whatever is the shape of the graph of $f(x)$, if you look at it under microscope in a particular point, it looks almost like straight line, almost like

$$f(x_0 + \Delta x) - f(x_0) \approx A_1\,\Delta x$$

This is why in many cases the small variations of variables in Physics (read: in Nature) enter all the laws and equations linearly, that is as Δx, not as $(\,\Delta x)^2$.

So: in many cases small changes of parameters cause linear feedback from the physical system….

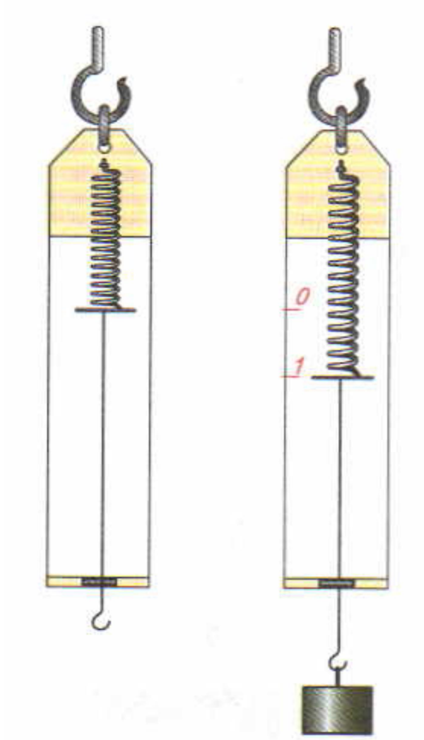


Supposedly, Hook's Law read when first time published like this: "CEIIINOSSSTTUV" … and only years later, after he was sure nobody else published it, Hooke ordered those letters in non-alphabetical order : "UT TENSIO, SIC VIS" (as extension, so is force):

$$\boldsymbol{F = -k\Delta x}$$

So the force is just proportional to the extension of spring (or any other elastic object!). The picture on the left (where $\Delta x = x$) will illustrate this. *k* is sometimes called *stiffness* of the spring, or *Hooke's coefficient.*

Hooke's law is used to measure force in simple dynamometers:



Homework problems.

(1) *A weight is hanging on a spring. It does not move, so the acceleration is zero. This implies that total force acting on it is zero. Discuss what forces make up the total force and how come they add up to zero.*

(2) *In SI force is measured in Newtons – Newton (symbol "N") is a force needed to accelerate 1 kg body with acceleration* $1\ m/s^2$. *In CGS units of force are Dynes (symbol "dyn"). Given that CGS uses centimeters, grams and seconds, try to make up definition of dyne and try to make relationship with Newtons – how many dyn is one N ?*