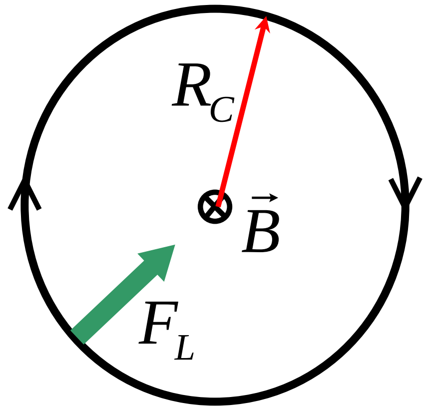
Homework 15.

Motion of a charged particle in a magnetic field.

Last class we discussed the motion of a charged particle in a magnetic field. Since the Lorentz force is perpendicular to the particle's velocity it does not do work and does not change the kinetic energy of the particle. It is only changes the direction of the particle's motion. If the particle's velocity is perpendicular to the magnetic induction ***B***, the path of the particle is a circle.

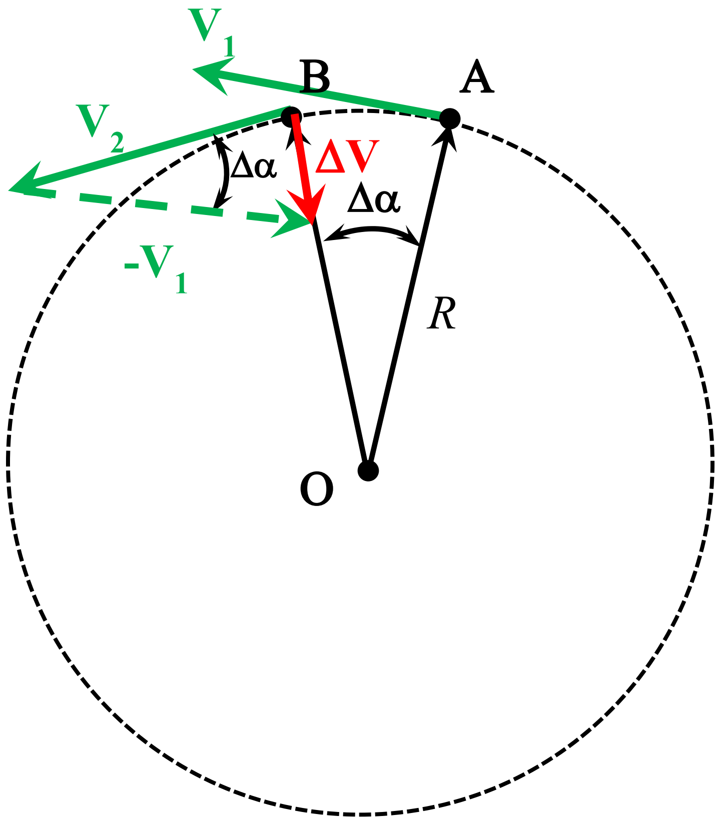


In Figure above, the green arrow shows the Lorentz force and magnetic induction ***B*** is directed "from us" perpendicular to the picture's plane. If we will apply our right-hand rule we see that the direction of the motion in the Figure corresponds to a negatively charged particle, say, an electron. Again, the Lorentz force does do the work on the particle since it is always directed perpendicular to the particle's displacement. As no work is done, the total energy of the particle (in this case it is just the kinetic energy) does not change and the *speed* of the particle remains constant. This type of circular motion is called *uniform* circular motion. Let us find how fast the particle will be revolving. First, let us write the expression of the second Newton's law for the particle:

$$ma = qVB \quad (1),$$

here $\boldsymbol{a}$, $\boldsymbol{m}$ and $\boldsymbol{q}$ are acceleration, mass and charge of the particle, $\boldsymbol{V}$ is its speed and B is the magnetic induction. In the equation (1) we used the projections of the force and acceleration to the radius drawn to the current position of the particle – that is why we do not put arrows over the vector quantities. For uniform circular motion the acceleration $\boldsymbol{a}$ is directed toward the center and is called *centripetal* acceleration. It can be calculated as: $\boldsymbol{a} = \boldsymbol{V}^2/\boldsymbol{R}_C$. This is not evident at all! You can take is as granted for a while or read the explanation below, if you want. If not, you can skip the italic text.

How to calculate the magnitude of the centripetal acceleration?



Imagine that an object is moving along a circle from point A to point B (see the picture above). The speed of the object is the same in both points (since it is uniform circular motion). The velocity vectors are different. As the object travels from point 1 to point 2, its velocity vector turns. We know that at the circular motion the velocity at any point is directed along the tangent line to this point. The tangent line is perpendicular to the radius drawn from the center to the "tangent" point. Simply speaking, each black arrow in the figure above is perpendicular to the corresponding green arrow. So, the angle ***Δα*** *between the velocities* $\boldsymbol{V_1}$ *and* $\boldsymbol{V_2}$ *is equal to the angle* ***Δα****, swept by the moving object.*

As we remember, acceleration is the change in velocity divided by the time, required for this change. Change in velocity ***ΔV*** *is shown by the red arrow in the picture above. To find it we have to subtract vector* ***V1*** *from the vector* ***V2****. To do that we will prepare the vector* ***–V1*** *which has the same length as* ***V1,*** *but its direction is opposite. Then we add* ***–V1*** *to* ***V2****.*

We have to find the length of the red arrow ΔV and divide it by the time ***Δt*** *which is required for the object to travel from point A to point B.*

$$a = \frac{\Delta V}{\Delta t} \quad (1a)$$

To calculate ΔV we assume that the arc AB is really really small, so the arc AB is very close to a straight line. We can see that

$$|AB| \approx R \cdot \Delta\alpha = V \cdot \Delta t \quad \textit{(2a)}$$

It follows from our way to measure the angle. It turns that the formula above is good for any "narrow" isosceles triangle! To find a "short" side we have to multiply one of the "long" sides to the small angle between them. The "narrower" the triangle, the more exact is the formula (2a). Let us apply this formula to the triangle formed by V_1, V_2 and ΔV:

$$\Delta V \approx V \cdot \Delta\alpha \qquad \textit{(3a)}$$

Let us plug ΔV from the formula (3a) to the formula (1):

$$a = \frac{\Delta V}{\Delta t} \approx \frac{V \cdot \Delta\alpha}{\Delta t} \qquad \textit{(4a)}$$

But $\frac{\Delta\alpha}{\Delta t} = \frac{V}{R}$ this follows from equation (2a). So we can write:

$$a = \frac{V^2}{R} \quad \textit{(5a),}$$

where R is the radius of the circle

We can plug this to the formula (1):

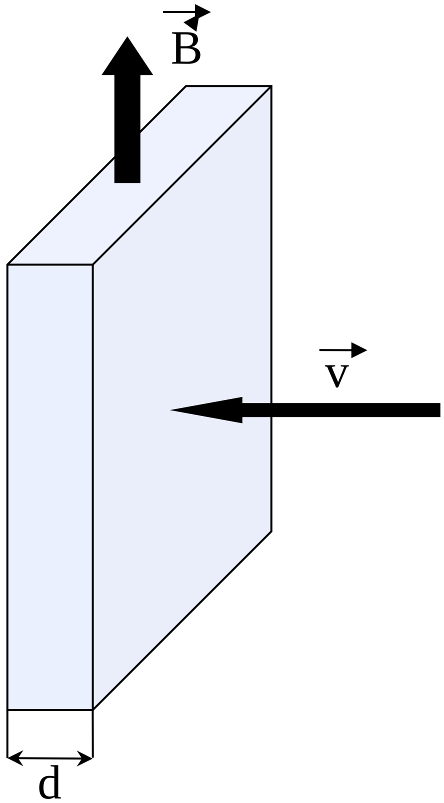
$$m\frac{V^2}{R_c} = qVB \quad (2)$$

Expressing V from the equation (2) we obtain:

$$V = \frac{eB}{m} R_c = \omega_c R_c \quad (3),$$

where $\omega_c = \frac{eB}{m}$ is the angular velocity (or angular frequency) of the moving charged particle. It is the angle, swept by the revolving particle per unit time and it is called ***cyclotron frequency***. It does not depend on the particle's velocity, so it can be used to measure the ratio of the particle's charge to its mass.

1. An electron enters the area with constant magnetic field **B** directed perpendicularly to the electron's velocity **v** (see picture below). The width of the area is ***d***.



a) What are possible trajectories of electron? (make a picture)
b) Find the expression for the electron's deflection angle.
c) Find the "critical" magnetic induction at which the electron can not cross the magnetic field area.

(You know electron's charge, mass, velocity)