Homework 16.

How does a current produce magnetic field. Biot-Savart-Laplace law.

We are going to discuss the source of magnetic field. As we learned before magnetic field is created by a moving charge. If a charge particle (or particles) is moving with respect to you, you can register magnetic field (if you have a suitable device). In other words, the source of magnetic field is current. Last class we started discussing how we can calculate the magnetic field created by a very short piece of wire. As long as you know the magnetic field created by a small straight piece of wire, we can find the magnetic field created by a wire loop of arbitrary shape, because we can represent this loop as a connection of small straight segments. We will discuss it in more details this Sunday.

Here am going to give you one example of application of this method – the magnetic field created by a infinitely long straight wire.

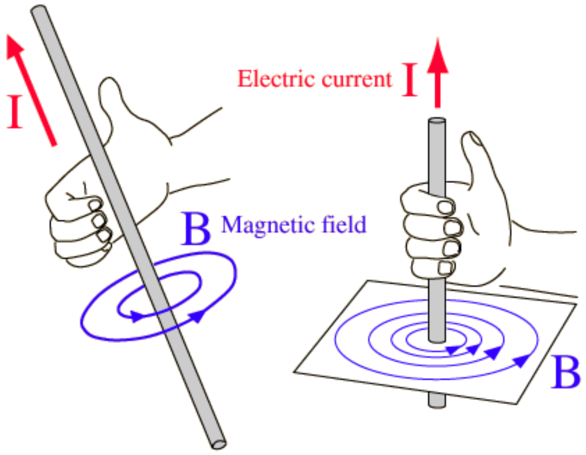

Figure 1.

Magnetic field is proportional to the inverse distance to the wire:

$$B(r) = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad (1)$$

Here I is the current in the wire, r is the distance to the wire and m0 is a constant which is called *magnetic constant* $\mu_0 = 4\pi \times 10^{-7}$ N/A^2.

Now we will discuss how it is possible to calculate the magnetic field created in a certain point (say, A) by an arbitrary shaped wire with current *I* (see Figure 2, left).

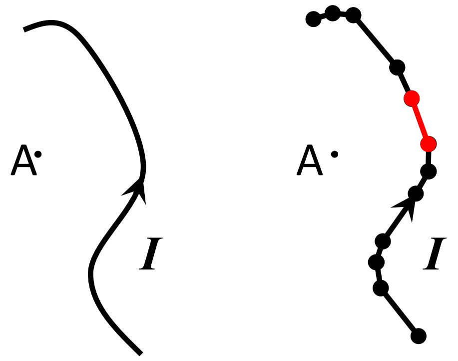

Figure 2.

To do that we could approximate the wire as a chain of very short straight segments (Figure 1, right). The total magnetic field B in point A is the vector sum of the magnetic fields ΔB created by each of the segment in point A (this is called "the superposition principle"). If we can calculate ΔB from each segment, then, in principle, the problem could be solved. In general case of an arbitrary shaped wire it is rather difficult and one will need a computer to perform the calculation. But, in some special cases, the calculation is really easy (as we will see later).

How to calculate ΔB? Let us chose an arbitrary segment of length ***Δl*** (shown in red in Figure 1, right). Then, let us draw the line connecting point A with the center of the segment. The vector from the center of the segment to point A is ***r***. We have to chose segments short enough, so ***Δl*** << **r**. Let us denote the angle between this line and the segment as ***α*** (Figure 2).

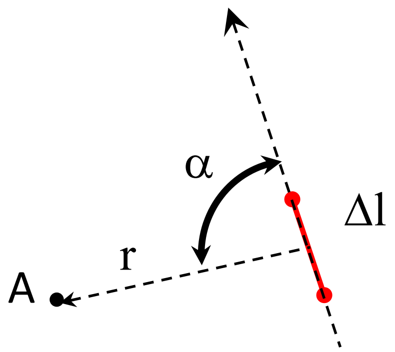

Figure 2.

Then, we can calculate ΔB using the following expression:

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\cdot\Delta l}{r^2} sin\alpha \quad (1)$$

In fact, formula 1 looks familiar. It can be written as already familiar to us cross product:

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I}{r^3}\left[\overrightarrow{\Delta l} \times \vec{r}\right] \quad (2)$$

Note that vector ***Δl*** is directed as the current. Formula (2) is called *Biot-Savart-Laplace* law. The direction of ΔB in point A we can find using the right hand rule. In general, all segments in Figure 1 have different length and orientation. In addition, the distances from each segment to point A are different. That is why it is generally a difficult problem. But, in some cases, as I mentioned, one can obtain the solution relatively easy – just following the procedure and using logic. As an example – the homework problem below:

Problem:

1. There are two parallel wires with current ***I*** (directed to the same side) and length ***L***. The distance between wires is ***R***. Find the formula for the force exerted by the wires to each other. Make a picture and show the direction of force.

2. Find magnetic field in a center of a round wire loop of radius ***R*** and with current ***I***.