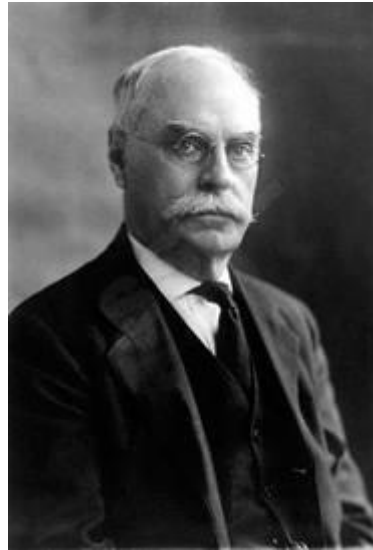


## Homework 18.

### Hall effect

During last class we discussed an important effect which has numerous technical applications such as position and motion sensors, automotive ignition, anti lock brake system (ABS) and many others. This effect is called Hall effect. It is named after Edwin Herbert Hall, American physicist who discovered the effect in 1879.



Edwin Herbert Hall (1855-1938)

I would say that the Hall effect is the direct consequence of the Lorentz force. Let us consider a rectangular piece of conducting material, say metal. As we know, metals are conductors and the electrical current in metals is carried by the electrons.

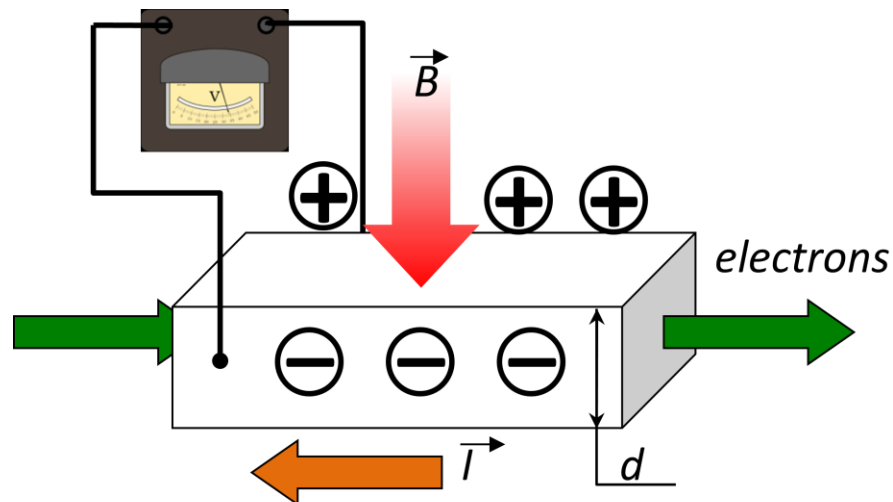


Figure 1. Hall effect illustration.

As we put the current through the material as shown in the Figure 1, and apply the magnetic field directed down, the voltage appears between the front and back facets of the piece (see the figure). The appearance of this voltage (which is called hall voltage) can be explained by the following way:

1. Let us consider what happens in the first moment after we have turned the current on (Figure 2, this is top view of our material).

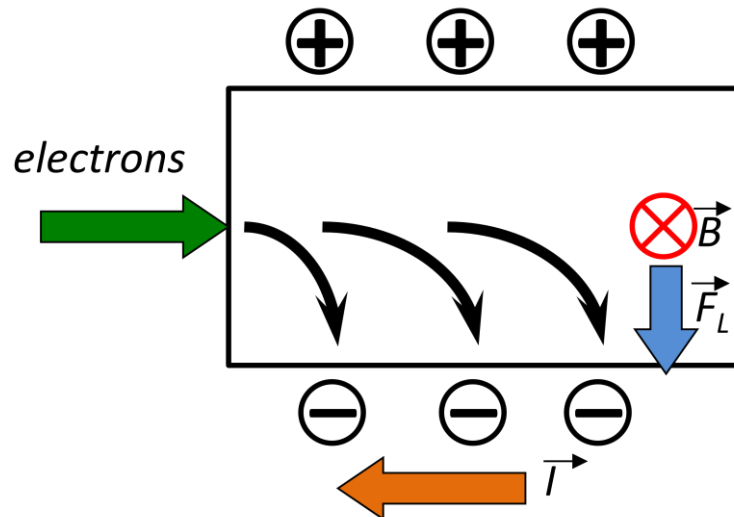


Figure 2. “Upper view” of the conductive parallelepiped immediately after the current is turned on.

The electrons will be deflected by the Lorentz force toward the front facet of the parallelepiped. So we will have excess electrons near the front facet and lack of electrons near the back one. As a result, the front facet is getting negatively charged, the back facet becomes “positive”. (You are encouraged to check the direction of the electron deflection using right-hand rule. Remember that the current direction is opposite to the direction of the electron flow.).

2. The electron will not be accumulating at the front facet forever. As long as the facets are getting charged the electric field appears inside the parallelepiped. This electric field applies the electric force to the electrons. This electric force compensates the Lorentz force and electrons are not deflected anymore. The current flows through the material as there is no magnetic field. But additional voltage  $V_H$  appears between the front and back facets of the parallelepiped (Figure 3).

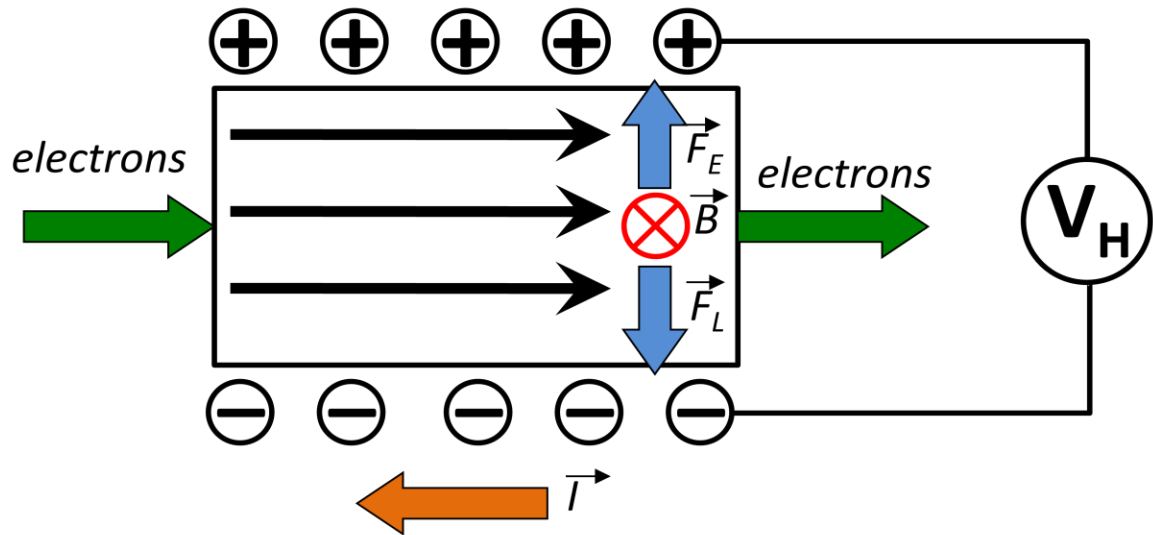


Figure 3. Steady state regime.

The absolute value of the Hall voltage can be calculated from the expression below:

$$V_H = \frac{I \cdot B}{e \cdot n \cdot d} \quad (1),$$

where  $d$  is the thickness,  $w$  is the width and  $L$  is the length of the parallelepiped;  $n$  is the electron density inside the parallelepiped (Here by the density we mean the number of electrons per cubic meter of the material),  $I$  is the current and  $B$  is the magnetic induction.

The rigorous theory of the Hall effect is more complicated. It considers the conductors of arbitrary shape, with several kinds of charge carriers inside. But the qualitative picture given above is a good description of the phenomenon.

Besides practical applications, the discovery of the Hall effect had a powerful scientific impact. The discoveries of the related phenomena called “Quantum hall Effect” and “Fractional Quantum Hall Effect” was awarded with Nobel prizes in Physics in 1985 (Klaus von Klitzing, Germany) and 1998 (Robert Laughlin, Horst Störmer and Daniel Tsui, USA).

### Problems

1. Assume that you have two pieces of different materials. The shape of the pieces is identical, but the electron concentration of the materials is different. Which material will show higher Hall voltage at the same current and magnetic induction? Give *physical* explanation to your answer (The explanations like “I used the formula” are not qualified as physical).
2. Explain the operation principle of a motion sensor based on the Hall effect.
3. *For the volunteers(if any 😊)*: Try to obtain formula (1). You will have to recollect the connection between electric field, voltage and electric force. Also try to think how the current relates to the velocity of the charge particles.