Oscillations

Many physical systems near equilibrium are describe by the following **Differential Equation**:

$$\ddot{x} = -\varpi^2 x$$

This is the second time derivative of ***x*** (acceleration)

By using analogy with rotation, we have found in class that solution to this equation is an oscillatory motion with period T=2π/ω:

Angular Frequency, ω =2π/T

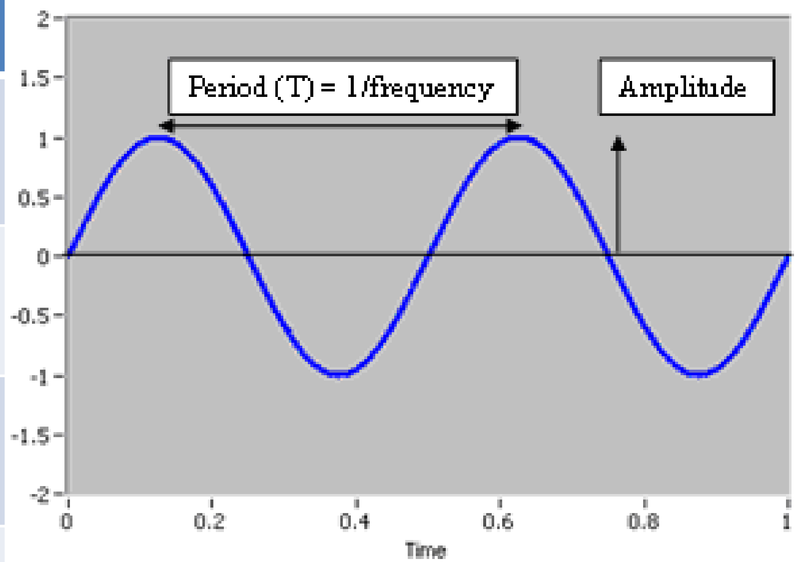
$$x(t) = A\sin\left(\varpi t + \varphi_0\right)$$

Amplitude

Phase Shift

$$T = \frac{1}{f} = \frac{2\pi}{\varpi}$$

Parameter	Formula	Units
Period	T	s
Frequency	f=1/T	1/s=Hz (Hertz)
Angular frequency	ω=2πf=2π/T	1/s
Amplitude	A	varies

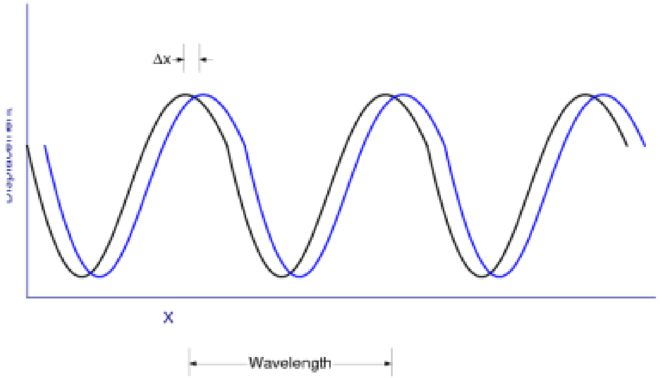


Travelling wave

$$x(t) = A\sin(\varpi t - kx) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

This wave moves to the positive direction of ***x*** with speed ***s***:

$$s = \frac{\lambda}{T} = \lambda f = \frac{\varpi}{k}$$



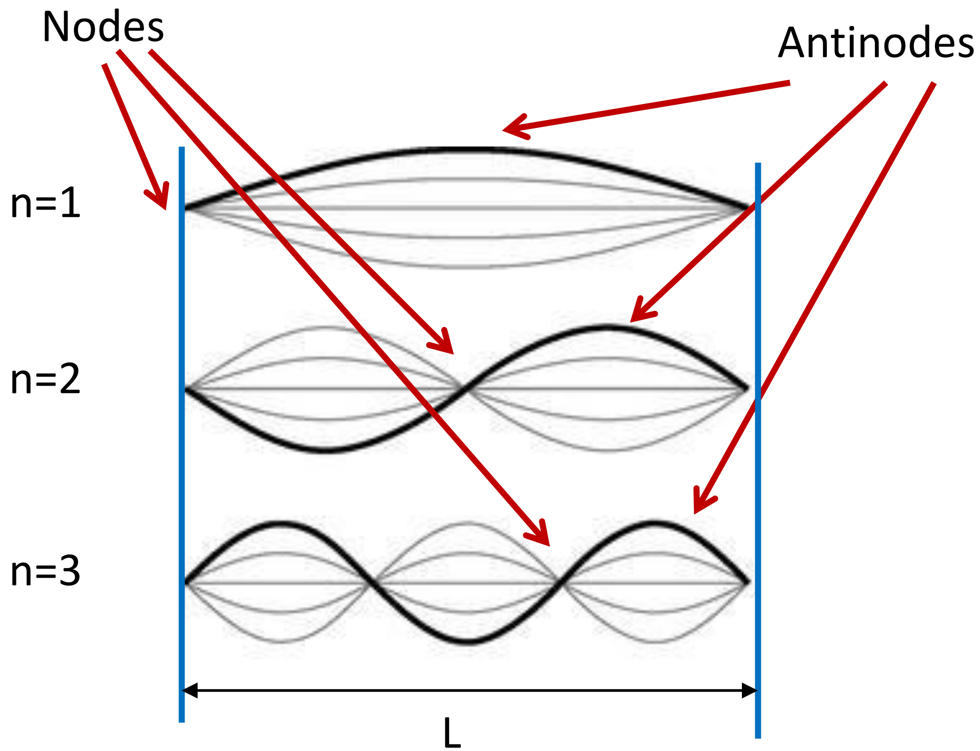
Oscillations	Wave
Period [s]: T	Wavelength[m]: λ
Angular frequency [1/s]: ω=2π/T	Wave Number [1/m]: k=2π/λ

Standing waves

$$A\sin(\varpi t - kx) + A\sin(\varpi t + kx) = 2A\sin(kx)\cos(\varpi t)$$

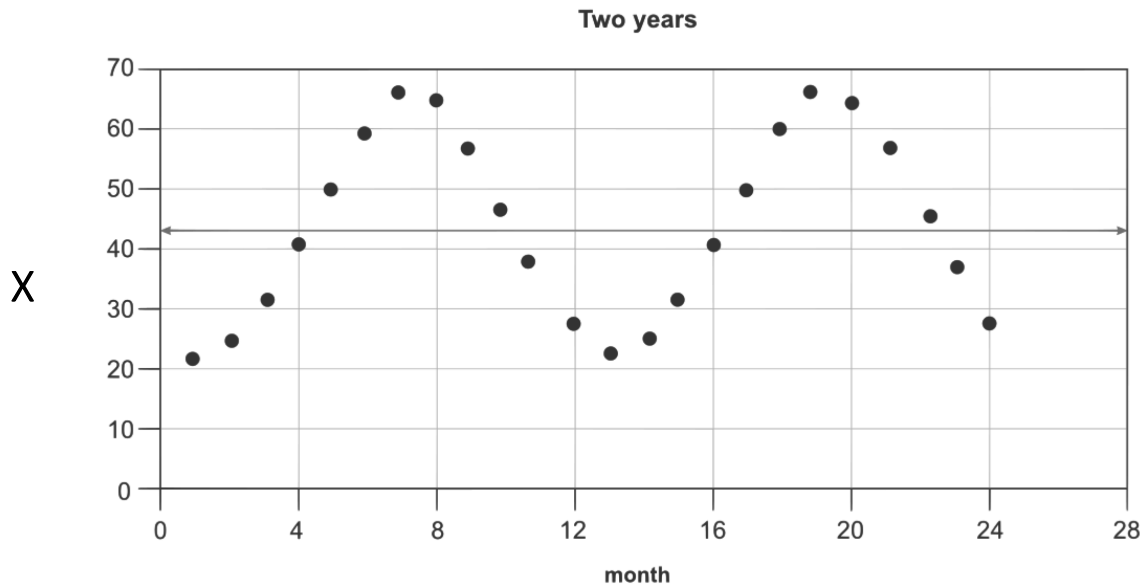
Wave moving in '+' direction + Wave moving in '-' direction = Standing Wave

$$\lambda = 2Ln, \qquad n = 1, 2, 3\ldots$$



Homework

Problem 1 Write a formula that would fit the plot x(t), shown below (t in months):



Problem 2

Consider a pipe of length 1 m, with both of its ends sealed. This pipe is a resonator in which one can excite standing sound waves.

a) What are the wavelengths of the first three of theses waves (i.e. the three longest).

b) Find the frequencies of these three sound waves. The speeds of sound in air is 330m/s