Homework 16

Lorentz transformations.

The result of Michelson-Morley experiment became clear after the work published by A. Einstein in 1905. In this work he demonstrated the constancy of the speed of light in all inertial reference frames.

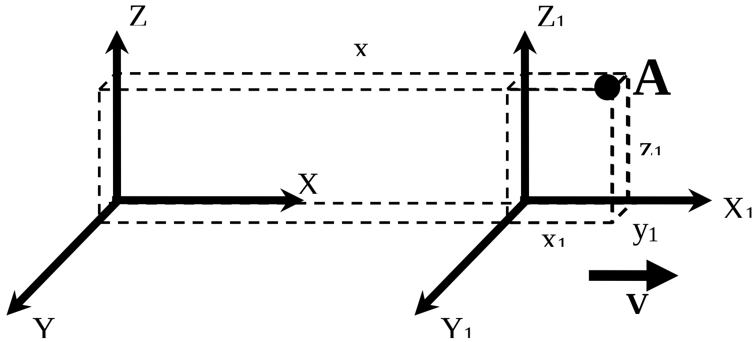

Fig.1

Let us consider a coordinate system $X_1Y_1Z_1$ which is moving at a velocity $\boldsymbol{V}$ with respect to the XYZ frame along the x axis (Fig.1). Let us imagine that the object A has coordinates x,y,z in the XYZ reference frame. We are going to express the coordinates x_1, y_1, z_1 of A in $X_1Y_1Z_1$ reference frame. According to classical nonrelativistic mechanics the transformation rules are:

$$\begin{aligned} x_1 &= x - Vt \\ y_1 &= y \\ z_1 &= z \\ t_1 &= t \end{aligned} \qquad (1)$$

The last equation looks absolutely trivial and intuitive. It means that the time flow is the same in both reference frames. However, the Michelson-Morley experiment suggested that the speed of light is the same in all inertial reference frames. This is in contradiction with the above coordinate transformations. Einstein demonstrated that the correct formulae are:

$$\begin{aligned} x_1 &= \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y_1 &= y \\ z_1 &= z \\ t_1 &= \frac{t - \frac{Vx}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{aligned} \qquad (2)$$

Here, c is the light speed in vacuum. These formulae – Lorentz transformations - were originally suggested by Hendrick Lorentz (1899) and, independently, by Joseph Larmor (1897).

Albert Einstein
(1879-1955).

The most interesting feature in the formulae (2) is that now the time “flow” depends on the velocity of the reference of frame. In a moving reference of frame the time “slowed down”. If you, using stopwatch, measure a certain period of time Δt (1 hour, say) and your classmate moving at a velocity V with respect to you will measure same time period using his or her stopwatch, you will notice that his (her) time period is longer:

$$\Delta t_1 = \frac{\Delta t}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (3)$$

You may think that now it is possible to say who is moving and who is staying – just check whose watch is slower. But, it does not work: your classmate will be convinced that it is *your* stopwatch which is slow. For him or her time is going normally. It is still not possible to determine who is moving and who is staying. So even if you are traveling in space at a very high speed and, for those who are left on the Earth your time is much slower than the Earth time, you will not enjoy extended life: in your reference frame time is “normal” as well as aging, unfortunately.
Same is for the length: the length of a moving object is reduced in the direction of motion (length contraction):

$$\Delta L_1 = \Delta L\sqrt{1-\frac{V^2}{c^2}} \qquad (34$$

And again this effect is only “seen from outside” – from a reference frame which moves with respect to yours at a speed V.

Problems:

In the problems below ***c*** is the speed of light in vacuum.

1. Imagine that an astronaut is moving at a speed 0.8c relative to the Earth. Find how long is his (her) hour as it is “seen” from the Earth?

2. Two events happen at the same time in a certain reference frame. Do you think that these events are simultaneous in all inertial reference frames? Proof your answer.

3. In a certain reference frame two events happen at different moments of time in different places. Is it possible to find another reference frame in which these events will happen at the same place?