Homework 19

Relativistic mass.

Last class we discussed relativistic mass. To start we consider a simple absolutely innelastic collision of two identical balls with a mass ***m*** mowing toward each other with identical speeds ***u*** (with respect to an observer1, Figure1). This approach is given in a nice book “Basic concepts in relativity and early quantum theory “ by Robert Resnick and David Halliday.

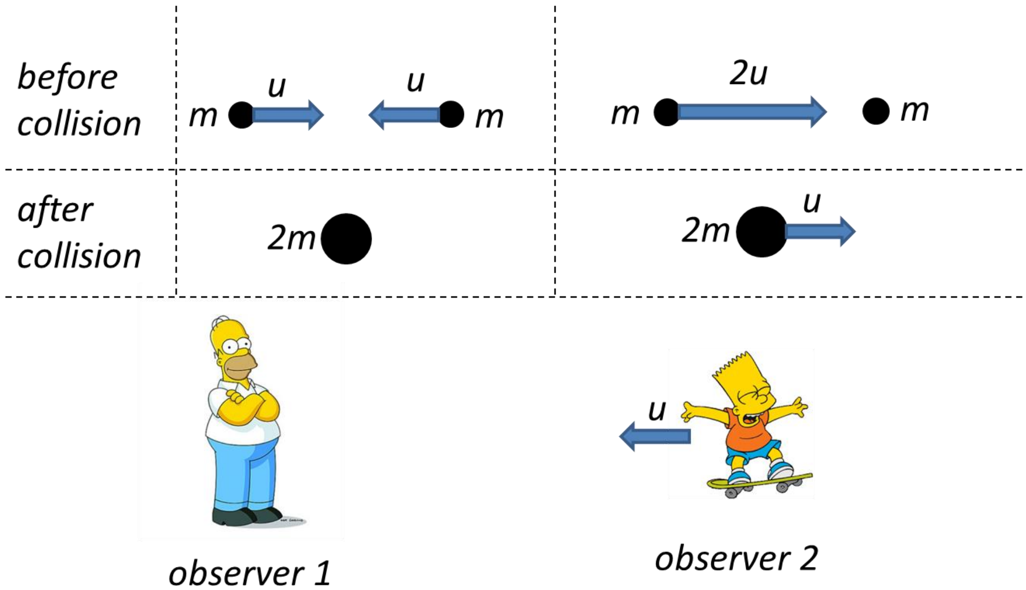

Figure 1.

After the collision the balls stick together and stop according to the momentum conservation law, stating that total momentum before collision (which is 0) has to be equal to the total momentum after collision. Observer 2, moving at a velocity u with respect to the observer 1 (see Figure 1) will see the ball moving at a velocity ***2u*** hitting another ball which is at rest. The total momentum of the balls before collision is ***2mu***, so after collision the big ball will be moving at a velocity ***u***. Everything looks fine at a first glance. And it *is* fine as long as observer 2 moves at a speed which is much less than the speed of light in vacuum ***c***. If this is not the case and ***u*** is comparable with ***c***, we cannot use the classic velocity composition rule and the speed of the left ball will not be measured as ***2u*** by the observer 2. Instead it will be :

$$u' = \frac{2u}{1+\frac{u^2}{c^2}} \qquad (1)$$

We can easily see that in this case the momentum conservation law seems to be broken for the observer 2. Instead of assuming that the momentum conservation law does not work, let us assume that the expression for classical momentum has to be changed for rapidly moving objects. As long as we know that the length and time intervals measured by observers moving with respect to each other are different (I mean time dilation and length contraction), it seems possible that the measured mass of the object depends on the velocity of the "measurer" with respect to the object. et us find this dependence.

Let us denote as $\boldsymbol{m_0}$ the mass of the particle in a reference frame where this particle is at rest; $\boldsymbol{m}$ will be the mass of a particle moving at a speed $\boldsymbol{u}$, and $\boldsymbol{m'}$ will be the mass of the particle moving at a speed $\boldsymbol{u'}$. (I used the word "speed" because we believe that due to symmetry considerations the mass of the particle does not depend on the direction of the velocity). The mass depending on the speed we will call *relativistic* mass. So, Figure 1 will be slightly changed.

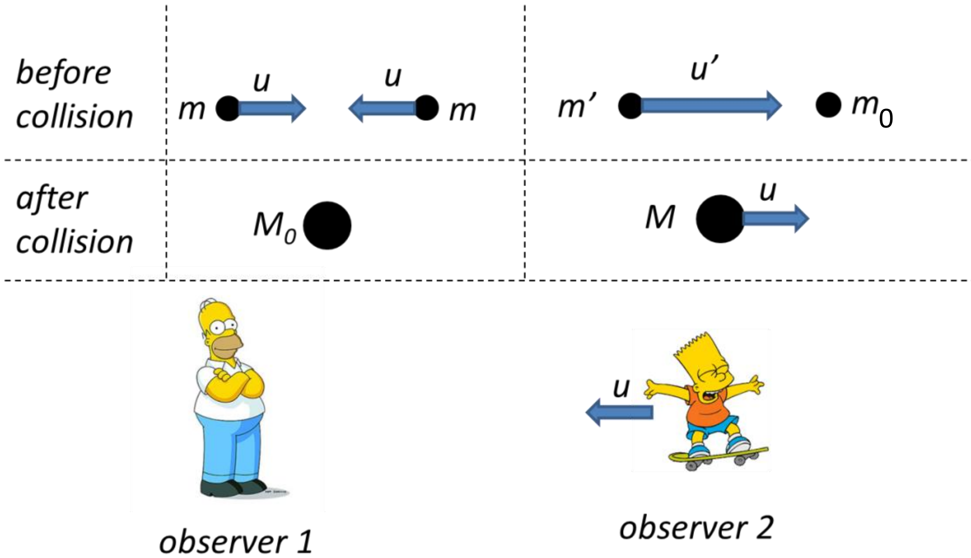

Figure 2.

Now we assume that correct formula for the momentum stays the same – the product of mass and velocity, but, instead of constant mass we will use the relativistic mass which depends on the speed of the object. This new momentum we, again, will call "relativistic". It is the relativistic momentum which conserves. For observer 2 the relativistic momentum conservation gives:

$$Mu = m'u' \qquad (2)$$

Next, we assume that relativistic mass conserves. Observer 2 will see that

$$M = m' + m_0 \qquad (3)$$

Now we have 3 equations (1,2 and 3 ☺) and five variables: $\boldsymbol{m_o, m', M, u, u'}$. To understand how does the relativistic mass depend on speed, we have to exclude $\boldsymbol{M}$ and $\boldsymbol{u}$ from the equations and obtain expression containing just $\boldsymbol{m_o, m',}$ and $\boldsymbol{u'}$. From (2) and (3) we obtain:

$$u = u'\left(\frac{m'}{m'+m_0}\right) \qquad (4)$$

After substitution u in the expression (1) using formula (4) we will obtain:

$$m' = \frac{m_0}{\sqrt{1-\frac{(u')^2}{c^2}}} \qquad (5)$$

So as long as an object having mass $\boldsymbol{m_0}$ in a rest reference frame is moving with respect to the observer with a speed $\boldsymbol{v}$, the mass of the object measured by the observer (relativistic mass) will be:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (6)$$

$\boldsymbol{m_0}$ is called rest mass. As long as $v \ll c$, then $m \approx m_0$ and, again, we can use expression of classical mechanics. As v approaching $\boldsymbol{c}$, the relativistic mass increases infinitely so it is impossible to accelerate an object with nonzero rest mass to the speed of light in vacuum.

Problems:

1. Find the speed at which the relativistic mass of the object is 2 times more than its rest mass.
2. You have 18g of water at temperature slightly higher than 0^0C. How does this mass change if you will heat it up to 100^0C? Give a rough estimate (take the kinetic energy of the water molecules as ~kT, where k is the Boltzmann constant, T is absolute temperature).