Homework 23

Black body radiation spectrum.

We have discussed the radiation spectrum of a black body. We learned that spectral radiant exitance of a black body can be expressed as:

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \qquad (1)$$

It looks a bit too complicated, but just for a first glance. Here ***h*** is the Planck's constant, **c** is the speed of light, $\boldsymbol{\lambda}$ is the wavelength, ***T*** is the temperature in absolute units (Kelvin scale), ***k*** is the Boltzmann constant ($k = 1.38 \cdot 10^{-23} J/K$) and ***e*** is mathematical constant , often referred as the Euler's number or Napier's number ($e \approx 2{,}718281828$ ....). As the famous P number, ***e*** is irrational so there is no period in the mantissa of ***e***.

What does the expression (1) mean? It shows the energy per unit wavelength, emitted from unit area of a black body at temperature T in a narrow wavelength range near the wavelength $\boldsymbol{\lambda}$. It shows the contribution of different wavelengths to the total power, emitted by a black body. The plots of the spectral radiant exitance of a black body at different temperatures are shown in Figure 1.

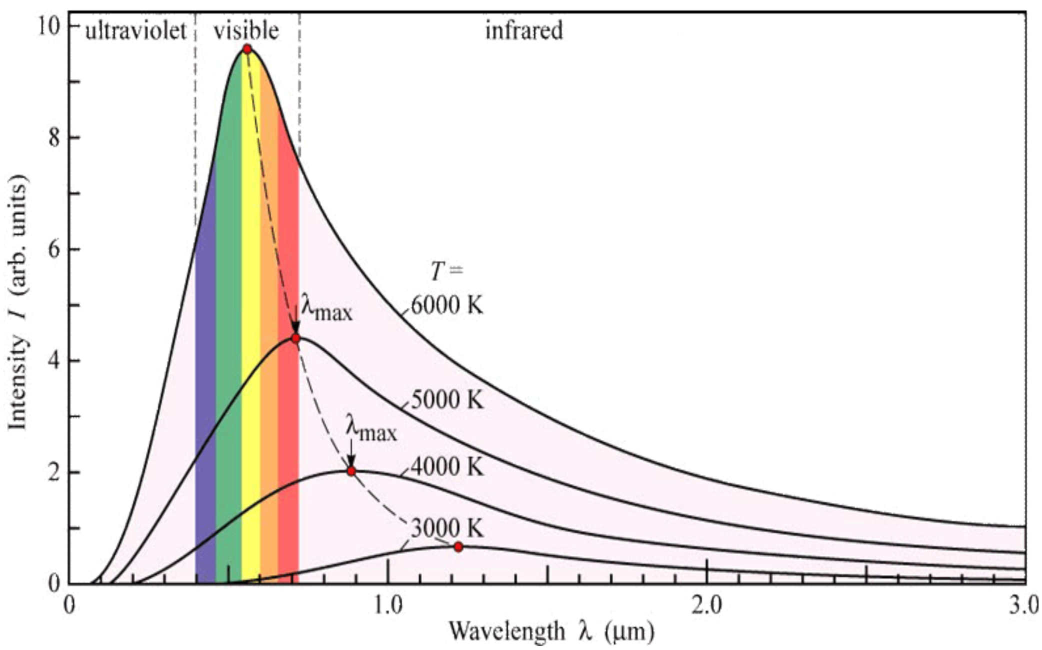

Figure 1. Schematic plots of the expression (1) at different temperatures. (the image is taken from http://www.globalchange.umich.edu/globalchange1/current/lectures/universe/universe.html)

Problems:

1. Find units of $R(\lambda, T)$ and explain the result.
2. Please simplify the expression (1) for the case of $\frac{hc}{\lambda kT} \ll 1$. For this one can use a following mathematical approximation: if $x \to 0$, then $e^x \approx 1 + x$.