Homework 8.

Unlike simple mechanical systems, gases can not be practically characterized by coordinates and velocities of each molecule – there is just not possible to perform calculations over billions of billions of molecules. Instead we can pick up some average parameters such as temperature T, pressure P and volume V (volume here is the volume of jar or bottle where the gas is kept). Pressure is measured in N/m^2 (Pa), Volume in m^3, Temperature in degrees according to Kelvin's scale (K). One Kelvin's degree equals to 1 Celsius degree, but zero at the Celsius scale is 273 degrees at the Kelvin's scale (or simply 273K). For example, room temperature is 27^oC, but 300K.

For ideal gas (we call gas "ideal" if the molecules do not interact with each other -they do not attract or repulse) T,P and V are connected by a simple equation:

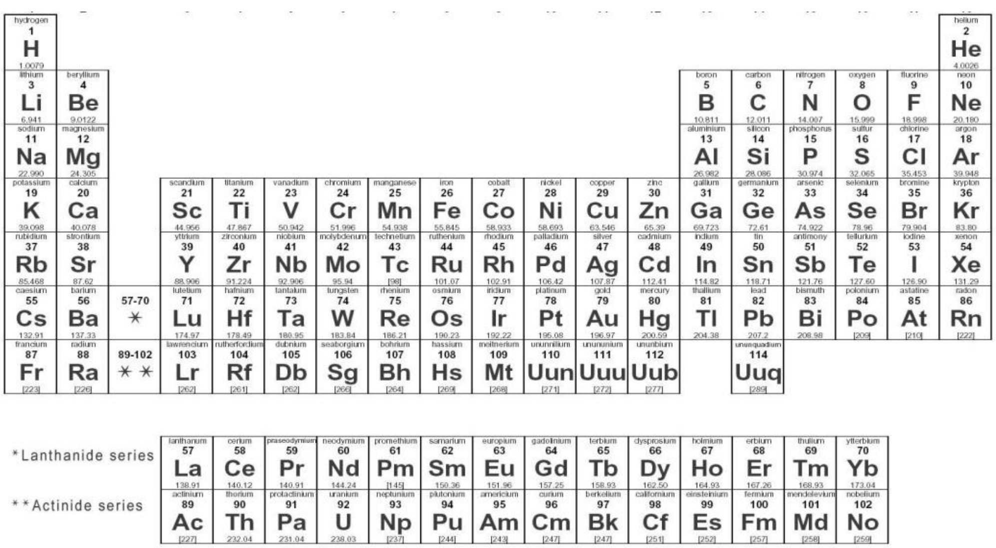
$$PV = nRT ,$$

where *n* is the number of molecules measured in *moles*, R is a constant which equals 8.31 J/(mole K), or $8.31x10^3$ J/(kmole K). One mole is a certain *number* of atoms or molecules. If we take ~$6x10^{23}$ molecules of, say, water we will have one mole of water ($6.02x10^{23}$ is called Avogadro's number). 1 kilomole (kmole) = 1000 moles.

How to find the number of moles (or kmoles) if we know a mass of a substance? First we have to find a mass of one molecule of the substance. To do that, we need to take a look into the periodic table of elements.

Periodic table of elements

Chemical elements are the "building blocks" of nature. All the objects around us are "constructed" from chemical elements. Despite great variety of the objects and substances around us there are only 118 chemical elements (some of them are not shown in the table below). They are systematized and arranged in the table which is called periodic table of elements.



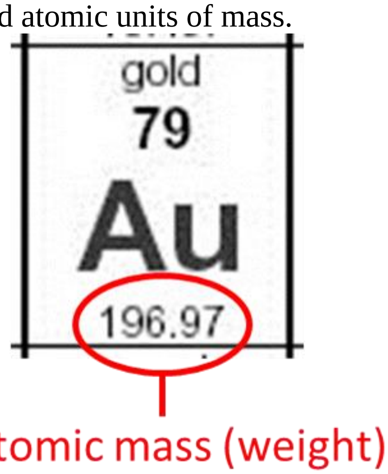
Periodic table of elements.

The periodic table was first suggested by a Russian chemist Dmitri Mendeleev in 1869. He found that if the chemical elements are arranged according their atomic mass, their chemical properties exhibit periodicity, that is why it is called "periodic".

Dmitri Mendeleev (1834-1907).

Only two of the chemical elements – mercury and bromine - are liquids at normal conditions (T=300K, atmospheric pressure), eleven elements are gases. The other elements are solids except nine elements (109-111 and 113-118) in the end of the table whose chemical properties are still unknown.

At the bottom of each cell of the table there is a number which represent the mass of the atom (atomic mass) in the so cold atomic units of mass.



One atomic unit of mass is 1.66×10^{-24}g, or 1.66×10^{-27}kg. It was chosen in such a way that if we take 1 mole of a substance (that means 6.02×10^{23} molecules), the mass of this 1 mole will be numerically equal to the atomic mass, but in grams.

For example, atomic mass of hydrogen (H) is ~1. A molecule of hydrogen consists of 2 atoms, so the mass of the molecule is 2 atomic units of mass. If we take 6.02×10^{23} molecules of hydrogen (1 mole), the total mass of the gas is 2g, or 0.002kg.

Another example: how many molecules (or moles) in 100g of water? A molecule of water consists of two atoms of hydrogen and one atom of oxygen. Let's look in the periodic table. Atomic mass of hydrogen is 1, atomic mass of oxygen is 16. So the mass of a molecule of water expressed in atomic units is 18. It means that a mass of 1 mole of water is 18g (we can say that *molecular mass* of water is 18g). Now we can easily find how many moles in 100g of water: 100g:18g/mole ~5.56moles. And we have total $5.56x6.02x10^{23}=33.44x10^{23}=3.34x10^{24}$.

Now, the problems (Please do not forget – we must use Kelvin's scale for temperature):

1. There is a 1 liter bottle filled with water at 27^oC. The water is liquid at this temperature because there is attracting force between the molecules. Imagine, that we have suddenly "turned off" this attracting force. What is the pressure in the bottle now?

2. What is the volume of 1 mole of an ideal gas at the temperature of 27^oC and pressure $105N/m^2$?

3. This problem is a bit more challenging: find the formula which express the density of an ideal gas through its molecular mass ($T=27^oC$, $P=105N/m^2$).