RELATIVITY OF MOTION

NOVEMBER 7, 2021

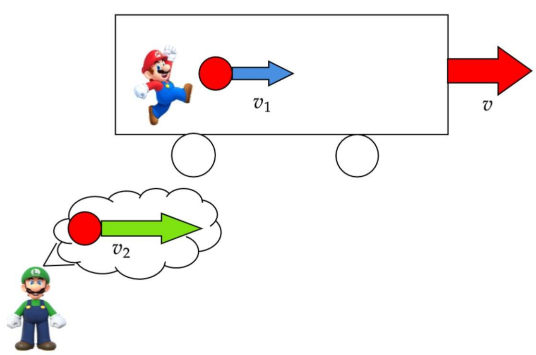
Theory recap

When we say that some object is moving, what do we really mean? Going in the car we see how trees along the road move, does it mean that the trees are moving? Intuitively, you could have said no - the car is burning gas in order to move, not the trees. But imagine that there is no friction and somebody has given your car a push. The car will go straight at constant speed, not burning any gas. Is it now so different from the trees that you see moving? You could still say yes - the trees are rooted in the ground, how could they move? But they actually can! They just move together with the roots and the ground.

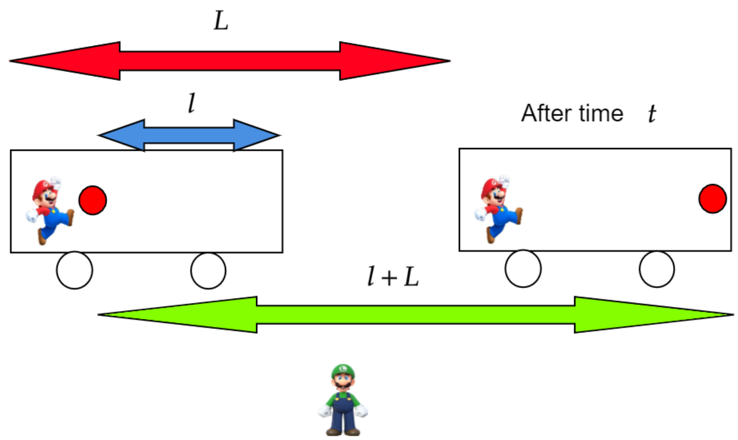
The idea of the whole ground moving may sound a little too crazy at first. Why not just admit that anything moving with respect to the ground actually moves and anything that stays motionless with respect to the ground actually stays at rest? It might be good on the Earth but what if we go into space? Suddenly we see how the Earth is moving 30 km/s around the Sun, and the Sun is moving around the center of our Milky Way galaxy and even our galaxy moves with respect to other galaxies! Where do we find something stationary in order to measure our movements?

The answer is, we will not find it. Not only it would have been impractical to relate all everyday movements to some cosmic object but there is simply no such object that is better than all the others. Every body in space or on Earth could be equally well used as a reference frame. When you stay near a road and say that cars move, this is correct as long as you mean that they move with respect to the you. When you sit in a car and say that trees and the ground are moving with respect to you, this is absolutely legitimate.

So, we could describe motion with respect to an arbitrary **reference frame**. In practice it could be useful to consider motion in different reference frames, so we need to learn how to switch between them. Consider the following situation: you are in a train carriage moving straight with constant speed v. You throw a ball with speed v_1 in the direction of train velocity. What would be ball speed v_2 with respect to your friend who stands outside the train?



To find speed, we need to divide distance by time. Suppose we measure the time it takes the ball to reach the front wall of the carriage. You and your friend will agree that it takes the same time t (you could both measure it with a stopwatch). But will you agree on the distance traveled by the ball? From your point of view the ball traveled distance l between you and the front wall. But your friend saw that additionally the carriage as a whole, with everything in it, traveled distance L in the same time t. Therefore the total distance traveled by the ball from their point of view is $l + L$ (see picture below).



Distance l traveled by the ball in carriage reference frame is $l = v_1 t$. The carriage itself travels $L = vt$. So our friend outside sees the ball moving by $l + L = v_1 t + vt = (v_1 + v)t$. Now we could find the speed of the ball with respect to the outside observer :

$$v_2 = \frac{l + L}{t} = \frac{(v_1 + v)t}{t} = v_1 + v$$

We have obtained a very important result: if you know velocity $\vec{v_1}$ of some object A in reference frame 1 and you know that the reference frame 1 moves with velocity $\vec{v}$ with respect to reference frame 2, then velocity of the object A in reference frame 2 is

$$\boxed{\vec{v_2} = \vec{v_1} + \vec{v}}$$

As this is a vector identity, remember to pay attention to the plus and minus signs, as we usually do.

Homework

1. The Earth moves around the Sun with speed 30 km/s. The Moon moves around the Earth with speed 1 km/s. Find maximal and minimal speed of the Moon with respect to the Sun. Draw a picture supporting your answer.
2. River flows with speed $v_r = 2\ m/s$. A fisherman uses his boat to get to a village situated at distance $d = 2\ km$ down the river, and returns back to his home. During the whole trip, the speed of the boat is $V = 3\ m/s$ with respect to the water. Find the total time of the two-way trip. Does river flow make it longer or shorter?

Below is a bonus problem

***3.** If an elastic ball hits a motionless wall at $90°$, its velocity just switches direction to the opposite, so speed stays the same. Now imagine that a ball with speed $v = 5\ m/s$ hits at $90°$ a wall moving with speed $u = 3\ m/s$ towards the ball. What will be speed of the ball after the collision? Why in tennis by hitting a ball moving towards him a player achieves a stronger shot (compared to hitting a non-moving ball)?